

# Calculus Pre/Post Placement Test Answer Key

I.

$$1. \quad f(g(x)) = f(3^x) = (3^x)^2 = 3^{2x}$$

$$f(g(2)) = 3^{2(2)} = 3^4 = 81$$

$$2. \quad f(x) + 2H(x) = x^2 + 2(1-x^2) \\ = x^2 + 2 - 2x^2 \\ = 2 - x^2$$

$$3. \quad \lim_{x \rightarrow 0} 3^x = 3^0 = 1$$

$$4. \quad \lim_{x \rightarrow \infty} \frac{H(x)}{f(x)} = \lim_{x \rightarrow \infty} \frac{1-x^2}{x^2} = \lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$$

$$5. \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 2x^2}{h} \\ = \lim_{h \rightarrow 0} \frac{(2x^2 + 2xh + h^2) - 2x^2}{h} \\ = \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} \\ = \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} \\ = \lim_{h \rightarrow 0} (4x + 2h) = 4x$$

II.

$$1. \quad r = \sqrt{1-2\theta} \quad \frac{dr}{d\theta} = \frac{1}{2}(1-2\theta)^{-\frac{1}{2}}(-2) \\ \frac{dr}{d\theta} = \frac{-1}{\sqrt{1-2\theta}}$$

$$2. \quad w = xe^{2x} \quad \frac{dw}{dx} = x(2e^{2x}) + e^{2x}(1) \\ = e^{2x}(2x+1)$$

$$3. \quad y = \ln(\sin(\theta)) \quad \frac{dy}{d\theta} = \frac{1}{\sin(\theta)} \cdot \cos(\theta) \\ = \cot(\theta)$$

III.

$$1. \quad \text{Let } u = \ln(x); \quad du = \frac{1}{x} dx$$

$$\int_1^{e^2} \frac{\sqrt{\ln(x)}}{x} dx = \int_1^{e^2} u^{\frac{1}{2}} du \\ = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \Big|_1^{e^2} \\ = \frac{2}{3} (\ln(x))^{\frac{3}{2}} \Big|_1^{e^2} \\ = \frac{2}{3} \left[ (\ln(e^2))^{\frac{3}{2}} - (\ln(1))^{\frac{3}{2}} \right] \\ = \frac{2}{3} (\sqrt{8}) \\ = \frac{2}{3} (2\sqrt{2}) = \frac{4\sqrt{2}}{3}$$

$$2. \quad \text{Let } u = \sin(2x); \quad du = \cos(2x) \cdot dx \cdot 2$$

$$\frac{1}{2} \int \sin(2x) \cos(2x) dx \cdot 2 \\ = \frac{1}{2} \int u du \\ = \frac{1}{2} \frac{u^2}{2} + C \\ = \frac{1}{4} (\sin^2(2x)) + C$$

$$IV. \quad f(x) = -2x^3 + 6x^2 - 3$$

$$f'(x) = -6x^2 + 12x$$

$$-6x(x-2) = 0$$

$x = 0, x = 2$  are critical points.

$$f''(x) = -12x + 12$$

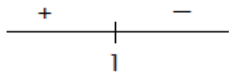
$$-12(x-1) = 0$$

$x = 1$  possible inflection point

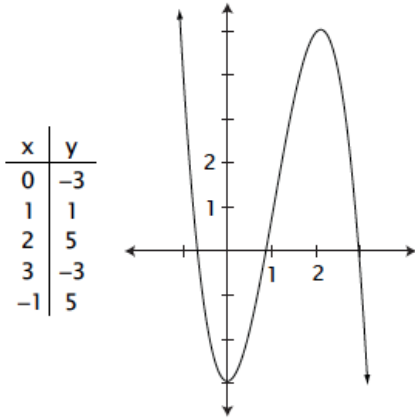
$$f''(0) = 12 \text{ so } x = 0 \text{ is a min.}$$

$$f''(2) = -12 \text{ so } x = 2 \text{ is a max.}$$

$f''$



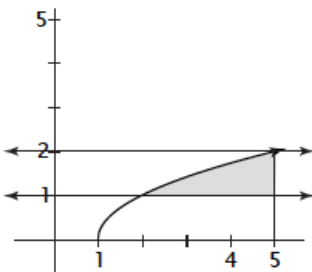
$x = 1$  is an inflection point



x	y
0	-3
1	1
2	5
3	-3
-1	5

$f$  is concave up from  $(-\infty, 1)$  and concave down from  $(1, \infty)$ .

V.  $y = \sqrt{x-1}$   
 $y^2 = x-1; x = y^2+1$



$$\int_1^2 (y^2+1) dy = \left( \frac{y^3}{3} + y \right) \Big|_1^2$$

$$= \left( \frac{8}{3} + 2 \right) - \left( \frac{1}{3} + 1 \right) = \frac{10}{3}$$

VI.

- $\lim_{x \rightarrow 3} \frac{x^2-9}{x^2+x} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x(x+1)} = 0$
- $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = \lim_{x \rightarrow 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = 6$
- $\lim_{x \rightarrow 0} \frac{\cos(x)-1}{\sin(x)} = \lim_{x \rightarrow 0} \frac{-\sin(x)}{\cos(x)} = 0$  using LR
- $\lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^{2x}$   
 $\frac{5}{x} \rightarrow 0$  so  $\lim_{x \rightarrow \infty} 1^{2x} = 1$
- $\lim_{x \rightarrow \infty} \frac{3x^2-4x+7}{x^2+3} = \lim_{x \rightarrow \infty} \frac{3 - \frac{4}{x} + \frac{7}{x^2}}{1 + \frac{3}{x^2}} = 3$

VII.  $f(x) = x^2 - \ln(x)$

$$f'(x) = 2x - \frac{1}{x}$$

$$f'(1) = 1$$

$$y = mx + b$$

$$1 = 1(1) + b$$

$$b = 0$$

So the equation of the tangent line

is  $y = x$ .

VIII.  $y = x^2 + 1; y = x + 3$

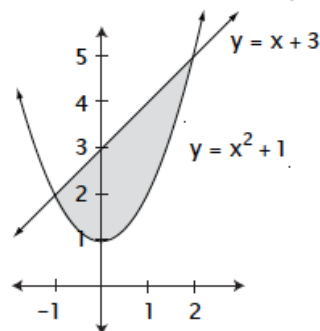
$$x^2 + 1 = x + 3$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1, 2$$

Points of intersection are  $(-1, 2)$  and  $(2, 5)$ .



$$\begin{aligned}
& \int_{-1}^2 [(x+3) - (x+1)] dx \\
&= \int_{-1}^2 (-x^2 + x + 2) dx \\
&= \left( -\frac{x^3}{3} + \frac{x^2}{2} + 2x \right) \Big|_{-1}^2 \\
&= \left( -\frac{8}{3} + 2 + 4 \right) - \left( \frac{1}{3} + \frac{1}{2} - 2 \right) \\
&= -\frac{8}{3} - \frac{5}{6} + 8 = 4\frac{1}{2}
\end{aligned}$$

IX.  $2x + 4y = 16$

$$y = 4 - \frac{1}{2}x$$

$$A(x) = xy$$

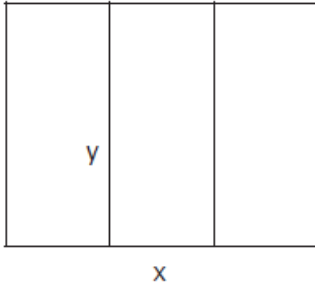
$$= x \left( 4 - \frac{1}{2}x \right)$$

$$= 4x - \frac{1}{2}x^2$$

$$A'(x) = 4 - x = 0 \quad \text{when } x = 4$$

$$A''(x) = -1$$

so  $A''(4) = -1$  which yields a max.



$$\text{If } x = 4, \text{ when } y = 4 - \frac{1}{2}(4) = 2.$$

The dimensions of the largest possible area would be 4 ft by 2 ft.