SCOPE & SEQUENCE

HONORS LESSON TOPICS

HOW TO USE MATH-U-SEE

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SCAPE & SEQUENCE

Math-U-See is a complete and comprehensive K-12 math curriculum. While each book focuses on a specific theme, Math-U-See continuously reviews and integrates topics and concepts presented in previous levels.

- **Primer**
- **α Alpha** | Focus: Single-Digit Addition and Subtraction
- **β Beta** | Focus: Multiple-Digit Addition and Subtraction
- **γ Gamma** | Focus: Multiplication
- **δ Delta** | Focus: Division
- **ε Epsilon** | Focus: Fractions
- **ζ Zeta** | Focus: Decimals and Percents

**Pre-Algebra**
**Algebra 1**
**Stewardship***
**Geometry**
**Algebra 2**
**Pre Calculus** With Trigonometry
**Calculus**

*Stewardship is a biblical approach to personal finance. The requisite knowledge for this curriculum is a mastery of the four basic operations, as well as fractions, decimals, and percents. In the Math-U-See sequence these topics are thoroughly covered in Alpha through Zeta. We also recommend Pre-Algebra and Algebra 1 since over half of the lessons require some knowledge of algebra. Stewardship may be studied as a one-year math course or in conjunction with any of the secondary math levels.*
HONORS LESSON TOPICS

Here are the topics for the special challenge lessons included in the student text. You will find one honors lesson at the end of each regular lesson. The instructions are on the honors pages in the student text.

**LESSON TOPIC**

01 Kinds of mathematics
02 Symmetry of graphs and equations
03 Interpolation
04 Factoring techniques
05 Using factoring techniques to simplify rational expressions

06 Trig applications
07 More trig applications
08 Bearings and headings
09 Navigation
10 More navigation

11 Proving cosine difference identity
12 Unit circle
13 Using trigonometry to find the area of a triangle
14 Rational roots test and the fundamental theorem of algebra
15 Synthetic division

16 More on the fundamental theorem of algebra
17 The remainder theorem
18 Vector applications - navigation and force
19 Graphs of common functions
20 Transformations of common functions

21 Inverse functions
22 More on natural log; the “most beautiful equation”
23 Continuous and discontinuous functions
24 Application of sine curve
25 Graphing calculators

26 Three-dimensional coordinate systems
27 Using series to find the natural log
28 Solving expressions with two inequality signs
29 Solving algebraic inequalities with quotients
30 Definition of the derivative
HOW TO USE MATH-U-SEE

Welcome to PreCalculus. I believe you will have a positive experience with the unique Math-U-See approach to teaching math. These first few pages explain the essence of this methodology which has worked for thousands of students and teachers. I hope you will take five minutes and read through these steps carefully.

If you are using the program properly and still need additional help, you may visit Math-U-See online at http://www.mathusee.com/support.html, or call us at 888-854-6284 (homeschools and individuals) or 800-454-6284 (schools and special education departments). —Steve Demme

The Goal of Math-U-See

The underlying assumption or premise of Math-U-See is that the reason we study math is to apply math in everyday situations. Our goal is to help produce confident problem solvers who enjoy the study of math. These are students who learn their math facts, rules, and formulas and are able to use this knowledge in solving word problems and real-life applications. Therefore, the study of math is much more than simply committing to memory a list of facts. It includes memorization, but it also encompasses learning underlying concepts that are critical to problem solving.

More than Memorization

Many people confuse memorization with understanding. Once while I was teaching seven junior high students, I asked how many pieces they would each receive if there were fourteen pieces. The students’ response was, “What do we do: add, subtract, multiply, or divide?” Knowing how to divide is important; understanding when to divide is equally important.
THE SUGGESTED 3-STEP MATH-U-SEE APPROACH

In order to train students to be confident problem solvers, here are the three steps that I suggest you use to get the most from the Math-U-See curriculum at this level:

Step 1. Preparation for the lesson.
Step 2. Presentation of the new topic.
Step 3. Progression after mastery.

Step 1. Preparation for the lesson.
This course assumes a knowledge of Geometry and Algebra 2. There are two review lessons in the appendix covering the Pythagorean theorem and special triangles. Watch the DVD to learn the concepts. Study the written explanations and examples in the instruction manual. Many students watch the DVD along with their instructor. Students in the secondary level who have taken responsibility to study this course themselves will do well to watch the DVD and read through the instruction manual.

Step 2. Presentation of the new topic.
Now that you have studied the new topic, choose problems from the instruction manual to present the new concept to your students.

a. Write: Record the step-by-step solutions on paper as you work them.
b. Say: Explain the “why” and “what” of the math as you work the problems.

Do as many problems as you feel are necessary until the student is comfortable with the new material. One of the joys of teaching is hearing a student say, “Now I get it!” or “Now I see it!”
Step 3. Progression after mastery.

Once mastery of the new concept is demonstrated, begin doing the pages in the student text for that lesson. Mastery can be demonstrated by having each student teach the new material back to you. The goal is not to fill in worksheets, but to be able to teach back what has been learned.

After the last student page in each lesson, you will find an "honors" lesson. These are optional, but highly recommended for students who will be taking advanced math or science courses. These challenging problems are a good way for all students to hone their problem-solving skills.

Proceed to the lesson tests. They were designed to be an assessment tool to help determine mastery, but they may also be used as extra worksheets. Your students will be ready for the next lesson only after demonstrating mastery of the new concept.

Confucius is reputed to have said, “Tell me, I forget; show me, I understand; let me do it, I will remember.” To which we add, “Let me teach it and I will have achieved mastery!”

Length of a Lesson

So how long should a lesson take? This will vary from student to student and from topic to topic. You may spend a day on a new topic, or you may spend several days. There are so many factors that influence this process that it is impossible to predict the length of time from one lesson to another. I have spent three days on a lesson, and I have also invested three weeks in a lesson. This occurred in the same book with the same student. If you move from lesson to lesson too quickly without the student demonstrating mastery, he will become overwhelmed and discouraged as he is exposed to more new material without having learned the previous topics. But if you move too slowly, your student may become bored and lose interest in math. I believe that as you regularly spend time working along with your student, you will sense when is the right time to take the test and progress through the book.

By following the three steps outlined above, you will have a much greater opportunity to succeed. Math must be taught sequentially, as it builds line upon line and precept upon precept on previously learned material. I hope you will try this methodology and move at your student’s pace. As you do, I think you will be helping to create a confident problem solver who enjoys the study of math.
Materials Needed

You will need the following items for this course:

- A protractor for measuring angles.
- A ruler with inches and/or metric measure.
- A scientific calculator that does square roots, trigonometric functions, logarithms, and natural log.

You can purchase an inexpensive calculator that does everything you need for this course. If you are planning on using it for more advanced courses, you can always get a more elaborate model.
Welcome to the Math-U-See Family!
Now that you have invested in your children’s education, I would like to tell you about the resources that are available to you. Allow me to introduce you to our staff, our ever improving website, the Math-U-See blog, our new free e-mail newsletter, and other online resources.

Many of our customer service representatives have been with us for over 10 years. What makes them unique is their desire to serve and their expertise. They are able to answer your questions, place your student(s) in the appropriate level, and provide knowledgeable support throughout the school year.

Come to your local curriculum fair where you can meet us face-to-face, see the latest products, attend a workshop, meet other MUS users at the booth, and be refreshed. We are at most curriculum fairs and events. To find the fair nearest you, click on “Events” under “E-sources.”

The Website, at www.MathUSee.com, is continually being updated and improved. It has many excellent tools to enhance your teaching and provide more practice for your student(s).

Math-U-See Blog
Interesting insights and up-to-date information appear regularly on the Math-U-See Blog. The blog features updates, rep highlights, fun pictures, and stories from other users. Visit us and get the latest scoop on what is happening.

Email Newsletter
For the latest news and practical teaching tips, sign up online for the free Math-U-See e-mail newsletter. Each month you will receive an e-mail with a teaching tip from Steve as well as the latest news from the website. It’s short, beneficial, and fun. Sign up today!
Online Support

You will find a variety of helpful tools on our website, including corrections lists, placement tests, answers to questions, and support options.

For Specific Math Help

When you have watched the DVD and read the instruction manual and still have a question, we are here to help. Call us or click the support link. Our trained staff are available to answer a question or walk you through a specific lesson.

Feedback

Send us an e-mail by clicking the feedback link. We are here to serve you and help you teach math. Ask a question, leave a comment, or tell us how you and your student are doing with Math-U-See.

Our hope and prayer is that you and your students will be equipped to have a successful experience with math!

Blessings,

Steve Demme
Introduction to Trigonometry

The word *trigonometry* comes from the Greek words τριγωνοσ (trigōnos), which means triangle, and μετρεο (metreo), which means to measure. The *Webster’s 1828 Dictionary* defines it as “the measuring of triangles; the science of determining the sides and angles of triangles, by means of certain parts which are given.”

From geometry, remember that a triangle is composed of three angles and three sides. Throughout this course, unless specified otherwise, we will be dealing with right triangles. We already know that one of the angles in a right triangle is 90° and that the side opposite the 90° angle is the *hypotenuse*.

Since we are dealing primarily with right triangles, the *Pythagorean theorem* also applies, so leg squared plus leg squared equals the hypotenuse squared. This is reviewed in Appendix A.

Special right triangles, such as the 30°– 60°– 90° and 45°– 45°– 90° triangles, pertain to our study as well. There is a brief refresher unit on this in Appendix B. Make sure the student understands the material in these two review lessons before proceeding further. They lay the foundation for our study of trigonometry.

In order to measure “the sides and angles of triangles by means of certain parts which are given,” as our definition tells us, we need to name the angles and sides. For example, in the 30°– 60°– 90° right triangle, the three sides are referred to as the short leg, the long leg, and the hypotenuse. Notice that the 30° angle (the smallest angle) is opposite the smaller leg, the 60° angle is opposite the longer leg, and the 90° right angle is opposite the hypotenuse. In the 45°– 45°– 90° triangle, the two legs are congruent, so we can’t describe them in terms of length. Let’s learn a new system of identification for trigonometry.

We need to describe all the angles and all the sides. Since this is a right triangle, we already know that one angle is 90° and the side opposite the right angle is the hypotenuse.
hypotenuse. This leaves two angles and two sides to name. I’ve decided to call the angles $\theta$ (theta) and $\alpha$ (alpha), both letters in the Greek alphabet. The sides will be described in reference to the angles. If I am standing in angle $\theta$ ($\angle \theta$), then the leg furthest away from me will be the \textit{opposite} side. The side that touches me (where my feet are standing in the triangle) is the \textit{adjacent} side.

If I move to stand in angle $\alpha$, then the side furthest away from me becomes the opposite side, and the side touching me is the adjacent side. The names for the sides depend on what angle you are referring to.

I’ll illustrate this with our old friend, the 3–4–5 right triangle.

![3-4-5 right triangle diagram]

Standing at $\theta$, the opposite side is three units long and the adjacent side is four units long. If I move to $\alpha$, then four is opposite and three is adjacent. In both instances, the hypotenuse is five units long.

Now we come to the six trigonometric ratios, using the terminology of opposite, adjacent, and hypotenuse. The three main ratios are \textit{sine}, \textit{cosine}, and \textit{tangent}. The sine of either angle in a right triangle (in our example $\theta$ or $\alpha$) is described as the ratio of the opposite side over the hypotenuse.

\[ \text{sine} = \frac{\text{opposite}}{\text{hypotenuse}} \]

Using the 3–4–5 right triangle, the sine of angle $\theta$ is shown below.

\[ \sin \theta = \frac{3}{5} \]

The cosine of $\theta$ is the adjacent over the hypotenuse, or \[ \cos \theta = \frac{4}{5}. \]

And the tangent of $\theta$ is the opposite over the adjacent, or \[ \tan \theta = \frac{3}{4}. \]
A fun way to remember these three trigonometric ratios is the result of dropping a brick on your big toe. What would you do? Probably get a pan of water and “soak a toe,” or SOH–CAH–TOA.

SOH stands for \( \sin = \frac{\text{opposite}}{\text{hypotenuse}} \Rightarrow S = \frac{O}{H} \)

CAH stands for \( \cos = \frac{\text{adjacent}}{\text{hypotenuse}} \Rightarrow C = \frac{A}{H} \)

TOA stands for \( \tan = \frac{\text{opposite}}{\text{adjacent}} \Rightarrow T = \frac{O}{A} \)

Another relationship found in these trigonometric expressions is that sine and cosine are complementary angles. In a right triangle (with one right angle), the other two angles always add up to 90°, so they are complementary.

Let’s look at the ratios again. They are constant for a 45°– 45°– 90° triangle or a 30°– 60°– 90° triangle. (See Appendix B.) In a 30°– 60°– 90° triangle, the short side is always one-half the hypotenuse. The length of the sides of all the 30°– 60°– 90° right triangles may vary, but the short side will always be one-half of the hypotenuse. Using our new terminology, we can now show it as follows.

\[
\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{short side}}{\text{hypotenuse}} = \frac{1}{2}
\]

In the next triangle notice that the short side, which is opposite 30°, is 5 unites long. The hypotenuse is 10 unites long. So the sine ratio is 5/10 or 1/2.
Here is another possibility.

\[ \sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{8} = \frac{1}{2} \]

Observe that the ratio of the small side to the hypotenuse remains constant.

**Example 1**
Find the sine, cosine, and tangent ratios for 30° and 60° in the triangle.

\[ \sin 30^\circ = \frac{5}{10} = \frac{1}{2} \]
\[ \sin 60^\circ = \frac{\sqrt{3}}{2} \]
\[ \cos 30^\circ = \frac{\sqrt{3}}{2} \]
\[ \cos 60^\circ = \frac{1}{2} \]
\[ \tan 30^\circ = \frac{5}{5\sqrt{3}} = \frac{\sqrt{3}}{3} \]
\[ \tan 60^\circ = \frac{\sqrt{3}}{3} \]

The ratios opp/hyp, adj/hyp, and opp/adj always depend on what angle is being referred to. That is why the sin 30° and the sin 60° are different, even though they are both opposite over hypotenuse.

**Example 2**
Find the sine, cosine, and tangent ratios for 45°. (Remember the relationships we found earlier between the lengths of the sides of a 45°-45°-90° triangle.)

\[ \sin 45^\circ = \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2} \]
\[ \cos 45^\circ = \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2} \]
\[ \tan 45^\circ = \frac{4}{4} = 1 \]

No matter what the lengths of the sides, the ratio for any given angle will always be the same when reduced to its simplest form.
It is very important that you memorize the fractional trig ratios for 30°, 45°, and 60° since they will be used in future lessons.

**Practice Problems 1**

Find the sine, cosine, and tangent of θ and α, and express the ratios in fraction form.

1. \( \sin \theta = \)  
2. \( \cos \theta = \)  
3. \( \tan \theta = \)  
4. \( \sin \alpha = \)  
5. \( \cos \alpha = \)  
6. \( \tan \alpha = \)  

7. \( \sin \theta = \)  
8. \( \cos \theta = \)  
9. \( \tan \theta = \)  
10. \( \sin \alpha = \)  
11. \( \cos \alpha = \)  
12. \( \tan \alpha = \)
13. \( \sin \theta = \)
14. \( \cos \theta = \)
15. \( \tan \theta = \)
16. \( \sin \alpha = \)
17. \( \cos \alpha = \)
18. \( \tan \alpha = \)

19. \( \sin \theta = \)
20. \( \cos \theta = \)
21. \( \tan \theta = \)
22. \( \sin \alpha = \)
23. \( \cos \alpha = \)
24. \( \tan \alpha = \)
Solutions 1

1. \( \sin \theta = \frac{5}{13} \)
2. \( \cos \theta = \frac{12}{13} \)
3. \( \tan \theta = \frac{5}{12} \)
4. \( \sin \alpha = \frac{12}{13} \)
5. \( \cos \alpha = \frac{5}{13} \)
6. \( \tan \alpha = \frac{12}{5} \) or \( 2 \frac{2}{5} \)

7. \( \sin \theta = \frac{7}{25} \)
8. \( \cos \theta = \frac{24}{25} \)
9. \( \tan \theta = \frac{7}{24} \)
10. \( \sin \alpha = \frac{24}{25} \)
11. \( \cos \alpha = \frac{7}{25} \)
12. \( \tan \alpha = \frac{24}{7} \) or \( 3 \frac{3}{7} \)

13. \( \sin \theta = \frac{6}{18} \) or \( \frac{1}{3} \)
14. \( \cos \theta = \frac{17}{18} \)
15. \( \tan \theta = \frac{6}{17} \)
16. \( \sin \alpha = \frac{17}{18} \)
17. \( \cos \alpha = \frac{6}{18} \) or \( \frac{1}{3} \)
18. \( \tan \alpha = \frac{17}{6} \) or \( 2 \frac{5}{6} \)

19. \( \sin \theta = \frac{8}{10} \) or \( \frac{4}{5} \)
20. \( \cos \theta = \frac{6}{10} \) or \( \frac{3}{5} \)
21. \( \tan \theta = \frac{8}{6} \) or \( 1 \frac{1}{3} \)
22. \( \sin \alpha = \frac{6}{10} \) or \( \frac{3}{5} \)
23. \( \cos \alpha = \frac{8}{10} \) or \( \frac{4}{5} \)
24. \( \tan \alpha = \frac{6}{8} \) or \( \frac{3}{4} \)
Reciprocal Trigonometric Ratios

There are three other basic trigonometric ratios in addition to sine = opp/hyp, cosine = adj/hyp, and tangent = opp/adj.

These ratios are the *reciprocals* of our original soh–cah–toa.

Where the sine (sin) = \( \frac{\text{opp}}{\text{hyp}} \), the *cosecant* (csc) = \( \frac{\text{hyp}}{\text{opp}} \).

Where the cosine (cos) = \( \frac{\text{adj}}{\text{hyp}} \), the *secant* (sec) = \( \frac{\text{hyp}}{\text{adj}} \).

Where tangent (tan) = \( \frac{\text{opp}}{\text{adj}} \), the *cotangent* (cot) = \( \frac{\text{adj}}{\text{opp}} \).

Now we have our stable full of all the possible trigonometric ratios. Study the following example to clarify the relationships between the ratios.

**Example 1**
Find all six trig ratios for \( \theta \).

\[
\begin{align*}
\sin \theta &= \frac{3}{5} & \text{csc} \theta &= \frac{5}{3} \\
\cos \theta &= \frac{4}{5} & \sec \theta &= \frac{5}{4} \\
\tan \theta &= \frac{3}{4} & \cot \theta &= \frac{4}{3}
\end{align*}
\]
Since the new ratios are reciprocals of the sine, cosine, and tangent, they can also be expressed as shown below.

\[
\begin{align*}
\csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta}
\end{align*}
\]

**Example 2**
Show that the relationships above are true using the ratios from example 1.

\[
\begin{align*}
\csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\
\frac{5}{3} &= \frac{1}{\frac{3}{5}} & \frac{5}{4} &= \frac{1}{\frac{4}{5}} & \frac{4}{3} &= \frac{1}{\frac{3}{4}} \\
\frac{5}{3} &= \frac{1 \times \frac{5}{3}}{\frac{3}{5} \times \frac{5}{3}} & \frac{5}{4} &= \frac{1 \times \frac{5}{4}}{\frac{4}{5} \times \frac{5}{4}} & \frac{4}{3} &= \frac{1 \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} \\
\frac{5}{3} &= \frac{5}{3} & \frac{5}{4} &= \frac{5}{4} & \frac{4}{3} &= \frac{4}{3}
\end{align*}
\]

**Practice Problems 1**
Find all six trig ratios for \( \theta \).

\[
\begin{align*}
\sin \theta &= & \csc \theta = \\
\cos \theta &= & \sec \theta = \\
\tan \theta &= & \cot \theta =
\end{align*}
\]
7. \( \sin \theta = \)  

8. \( \cos \theta = \)  

9. \( \tan \theta = \)

10. \( \csc \theta = \)

11. \( \sec \theta = \)

12. \( \cot \theta = \)

13. \( \sin \theta = \)

14. \( \cos \theta = \)

15. \( \tan \theta = \)

16. \( \csc \theta = \)

17. \( \sec \theta = \)

18. \( \cot \theta = \)

The other new concept in this lesson involves changing the fractions to decimals rounded to the ten-thousandths place. Look at the solutions for #1–6, and then go back and complete #7–18 by changing each fraction to a decimal.
Solutions 1

Here are #1–6 written as fractions and as decimals.

1. \( \sin \theta = \frac{6}{10} = .6000 \)
2. \( \cos \theta = \frac{8}{10} = .8000 \)
3. \( \tan \theta = \frac{6}{8} = .7500 \)
4. \( \csc \theta = \frac{10}{6} = 1.6667 \)
5. \( \sec \theta = \frac{10}{8} = 1.2500 \)
6. \( \cot \theta = \frac{8}{6} = 1.3333 \)

Here are #7–18 written as fractions and as decimals.

7. \( \sin \theta = \frac{24}{26} = .9231 \)
8. \( \cos \theta = \frac{10}{26} = .3846 \)
9. \( \tan \theta = \frac{24}{10} = 2.400 \)
10. \( \csc \theta = \frac{26}{24} = 1.0833 \)
11. \( \sec \theta = \frac{26}{10} = 2.6000 \)
12. \( \cot \theta = \frac{10}{24} = .4167 \)
13. \( \sin \theta = \frac{4}{8.5} = .4706 \)
14. \( \cos \theta = \frac{7.5}{8.5} = .8824 \)
15. \( \tan \theta = \frac{4}{7.5} = .5333 \)
16. \( \csc \theta = \frac{8.5}{4} = 2.1250 \)
17. \( \sec \theta = \frac{8.5}{7.5} = 1.1333 \)
18. \( \cot \theta = \frac{7.5}{4} = 1.8750 \)
Practice Problems 2
Find the missing side, and then name all six trigonometric ratios of \( \theta \) for each triangle. State your answers as fractions and as decimals. If an answer has a radical in it, leave it as a radical.

1.   

2.   

3.   

4.   

5. \[ \theta \]

6. \[ \theta \]

7. \[ \theta \]

8. \[ \theta \]

9. \[ \theta \]
### Solutions 2

1. \(3^2 + x^2 = 5^2\)
   - \(9 + x^2 = 25\)
   - \(x^2 = 16\)
   - \(x = 4\)
   \[
   \sin \theta = \frac{4}{5} = .8000,
   \csc \theta = \frac{5}{4} = 1.2500
   \]
   \[
   \cos \theta = \frac{3}{5} = .6000,
   \sec \theta = \frac{5}{3} = 1.6667
   \]
   \[
   \tan \theta = \frac{4}{3} = 1.3333,
   \cot \theta = \frac{3}{4} = .7500
   \]

2. \(5^2 + 12^2 = H^2\)
   - \(25 + 144 = H^2\)
   - \(169 = H^2\)
   - \(13 = H\)
   \[
   \sin \theta = \frac{12}{13} = .9231,
   \csc \theta = \frac{13}{12} = 1.0833
   \]
   \[
   \cos \theta = \frac{5}{13} = .3846,
   \sec \theta = \frac{13}{5} = 2.6000
   \]
   \[
   \tan \theta = \frac{12}{5} = 2.4000,
   \cot \theta = \frac{5}{12} = .4167
   \]

3. \(15^2 + x^2 = 17^2\)
   - \(x^2 = 64\)
   - \(x = 8\)
   \[
   \sin \theta = \frac{15}{17} = .8824,
   \csc \theta = \frac{17}{15} = 1.1333
   \]
   \[
   \cos \theta = \frac{8}{17} = .4706,
   \sec \theta = \frac{17}{8} = 2.1250
   \]
   \[
   \tan \theta = \frac{15}{8} = 1.8750,
   \cot \theta = \frac{8}{15} = .5333
   \]

4. \(8^2 + 10^2 = H^2\)
   - \(64 + 100 = H^2\)
   - \(164 = H^2\)
   - \(2\sqrt{41} = H\)
   \[
   \sin \theta = \frac{10}{2\sqrt{41}} = \frac{5\sqrt{41}}{41},
   \csc \theta = \frac{2\sqrt{41}}{10} = \frac{\sqrt{41}}{5}
   \]
   \[
   \cos \theta = \frac{8}{2\sqrt{41}} = \frac{4\sqrt{41}}{41},
   \sec \theta = \frac{2\sqrt{41}}{8} = \frac{\sqrt{41}}{4}
   \]
   \[
   \tan \theta = \frac{10}{8} = 1.2500,
   \cot \theta = \frac{8}{10} = .8000
   \]

5. \(24^2 + x^2 = 25^2\)
   - \(x^2 = 49\)
   - \(x = 7\)
   \[
   \sin \theta = \frac{24}{25} = .9600,
   \csc \theta = \frac{25}{24} = 1.0417
   \]
   \[
   \cos \theta = \frac{7}{25} = .2800,
   \sec \theta = \frac{25}{7} = 3.5714
   \]
   \[
   \tan \theta = \frac{24}{7} = 3.4286,
   \cot \theta = \frac{7}{24} = .2917
   \]
### Lesson 2 - Reciprocal Trigonometric Ratios

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<th>Expression</th>
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<th>( \cos \theta )</th>
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<th>( \sec \theta )</th>
<th>( \cot \theta )</th>
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Interpreting the Trigonometry Tables

In this lesson, we’ll work on reading and understanding the trigonometry tables. Remember that a trigonometric ratio may be represented as a fraction, or ratio. The fraction can then be transformed to a decimal by dividing the denominator into the numerator. Trigonometry (or trig) tables are simply lists of ratios or fractions expressed as decimals. Turn the page to find a table of trig ratios. These ratios may also be referred to as trigonometric functions.

To find the trig ratio when the measure of the angle is given, move down the first column to find the angle measure. The angle measures are listed in degrees. Then choose the appropriate column from the three columns which represent the trig functions: sine, cosine, or tangent. Now read the decimal ratio.

**Example 1**

Find the decimal ratio for cos 43°.

Locate 43° in the first column and move across to the cosine column. The decimal ratio is .7314.

**Practice Problems 1**

Using the trig table, find the ratios for the following trigonometric ratios, or functions.

1. sin 32° = 2. cos 46° =

3. tan 15° = 4. tan 45° =
5. \( \sin 60^\circ = \)  
6. \( \tan 27^\circ = \)

7. \( \cos 58^\circ = \)  
8. \( \sin 0^\circ = \)

9. \( \cos 73^\circ = \)

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<td>1. ( \sin 32^\circ = .5299 )</td>
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<td>3. ( \tan 15^\circ = .2679 )</td>
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<td>5. ( \sin 60^\circ = .8660 )</td>
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<td>7. ( \cos 58^\circ = .5299 )</td>
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<td>9. ( \cos 73^\circ = .2924 )</td>
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You’ve probably observed that the ratios are the same when they are computed from two complementary angles. In a 3–4–5 right triangle, the sine ratio for \( \theta \) is 3/5, which is also the cosine ratio for \( \alpha \). Since \( \sin \theta = 3/5 \) and \( \cos \alpha = 3/5 \), we see that \( \sin \theta = \cos \alpha \). Since this is a right triangle, \( \theta \) and \( \alpha \) are complementary angles (they add up to 90°).

The trig table may also be used to find the measure of an angle when the ratio is known. Choose the appropriate column (sine, cosine, or tangent), look down the column to find the four-digit decimal which is closest to the ratio given, and then read across to find the measure of the angle.

**Example 2**

Given the ratio .7813 for the tangent, find the measure of the angle.
Locate .7813 in the third (tangent) column, and then move left to the degrees in the angle column.

The measure of the angle is 38°.
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Practice Problems 2
Using the trig table, find the angle measure that corresponds to the ratio for each of the following trigonometric functions.

1. \( \tan \underline{\theta}^\circ = 1.0724 \)
2. \( \cos \underline{\theta}^\circ = .9063 \)
3. \( \tan \underline{\theta}^\circ = 2.7475 \)
4. \( \tan \underline{\theta}^\circ = .1944 \)
5. \( \sin \underline{\theta}^\circ = .1564 \)
6. \( \sin \underline{\theta}^\circ = .8290 \)
7. \( \cos \underline{\theta}^\circ = .6157 \)
8. \( \sin \underline{\theta}^\circ = .5592 \)
9. \( \cos \underline{\theta}^\circ = .1564 \)

Solutions 2

1. \( \tan 47^\circ = 1.0724 \)
2. \( \cos 25^\circ = .9063 \)
3. \( \tan 70^\circ = 2.7475 \)
4. \( \tan 11^\circ = .1944 \)
5. \( \sin 9^\circ = .1564 \)
6. \( \sin 56^\circ = .8290 \)
7. \( \cos 52^\circ = .6157 \)
8. \( \sin 34^\circ = .5592 \)
9. \( \cos 81^\circ = .1564 \)

Let’s take it a step further and find the decimal ratio by studying the 30° angle in a right triangle. \( \sin 30^\circ = .5000 \), which is another way of saying that the ratio of the small side (opposite the small angle) to the hypotenuse is 1 to 2, or 1/2, or .5 or .5000. Notice that \( \cos 60^\circ \) is also 1/2 or .5000.

As an example, let’s use the trig ratios found in the table to discover the measures of the angles in the 3–4–5 right triangle. These angle measures will be approximate since none of the ratios exactly match our results when we change the fractions to decimals.
Example 3
Find the measure of angle $\theta$ to the nearest degree using at least two trig functions to confirm your answer.

Figure 1

$\sin \theta = \frac{3}{5} = .6000$.
Looking in the table in the sine column, we see that this is closest to .6018 which indicates $37^\circ$.

$\cos \theta = \frac{4}{5} = .8000$, which is closest to .7986 in the cosine column, indicating and confirming $37^\circ$.

Example 4
Using the same triangle, find the measure of angle $\theta$ to the nearest degree. Use at least two trig functions to confirm your answer.

To find $\alpha$, we can choose from any of these three ratios:

$\sin \alpha = \frac{4}{5} = .8000$

$\cos \alpha = \frac{3}{5} = .6000$

$\tan \alpha = \frac{4}{3} \approx 1.3333$

These ratios all indicate $53^\circ$ for $\alpha$. Since $\alpha$ and $\theta$ are complementary angles, we can also confirm this by $90^\circ - 37^\circ = 53^\circ$. 
The tangent may be expressed as the sine over the cosine as well as opposite over the adjacent. Since the sine is the opposite over the hypotenuse and the cosine is the adjacent over the hypotenuse, if you put them over each other, the hypotenuse is canceled and you are left with the opposite over the adjacent. We’ll work this out using the triangle in example 4.

\[
\begin{align*}
\sin 37^\circ &= \frac{3}{5} \\
\cos 37^\circ &= \frac{4}{5} \\
\tan 37^\circ &= \frac{\sin 37^\circ}{\cos 37^\circ} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4} = \frac{\text{opposite}}{\text{adjacent}}
\end{align*}
\]

This relationship will be used in upcoming lessons, but I wanted you to notice it now. Conversely, the cotangent may be expressed as cosine over the sine.

\[
\tan = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin}{\cos} \quad \cot = \frac{\text{adjacent}}{\text{opposite}} = \frac{\cos}{\sin}
\]

**Example 5**
Find the measures of the angles in the following triangle, showing all three ratios for each angle.

\[
\begin{align*}
\sin \theta &= 6/20 = .3000 \\
\sin \alpha &= 19/20 = .9500 \\
\cos \theta &= 19/20 = .9500 \\
\cos \alpha &= 6/20 = .3000 \\
\tan \theta &= 6/19 \approx .3158 \\
\tan \alpha &= 19/6 \approx 3.1667 \\
\text{Using the table, } \theta &= 18^\circ \\
\text{Using the table, } \alpha &= 72^\circ
\end{align*}
\]
Practice Problems 3
There are three ways to find the measure of an angle: sin, cos, and tan. In each of these problems, use all three ratios to confirm the measures of angle $\theta$ and angle $\alpha$.

1. \[20\]
   \[\theta \quad 5.8\]
   \[19.1\]

   \[
   \begin{align*}
   \sin \theta &= \_ \_ \_ \\
   \cos \theta &= \_ \_ \_ \\
   \tan \theta &= \_ \_ \_ \\
   \theta &= \_ \_ ^\circ
   \end{align*}
   \]

   \[
   \begin{align*}
   \sin \alpha &= \_ \_ \_ \\
   \cos \alpha &= \_ \_ \_ \\
   \tan \alpha &= \_ \_ \_ \\
   \alpha &= \_ \_ ^\circ
   \end{align*}
   \]

2. \[38\]
   \[\alpha \quad 23.4\]
   \[30\]

   \[
   \begin{align*}
   \sin \theta &= \_ \_ \_ \\
   \cos \theta &= \_ \_ \_ \\
   \tan \theta &= \_ \_ \_ \\
   \theta &= \_ \_ ^\circ
   \end{align*}
   \]

   \[
   \begin{align*}
   \sin \alpha &= \_ \_ \_ \\
   \cos \alpha &= \_ \_ \_ \\
   \tan \alpha &= \_ \_ \_ \\
   \alpha &= \_ \_ ^\circ
   \end{align*}
   \]
3.

\[ \theta = \ldots^\circ \quad \alpha = \ldots^\circ \]

\[ \sin \theta = \ldots \quad \sin \alpha = \ldots \]
\[ \cos \theta = \ldots \quad \cos \alpha = \ldots \]
\[ \tan \theta = \ldots \quad \tan \alpha = \ldots \]

4.

\[ \theta = \ldots^\circ \quad \alpha = \ldots^\circ \]

\[ \sin \theta = \ldots \quad \sin \alpha = \ldots \]
\[ \cos \theta = \ldots \quad \cos \alpha = \ldots \]
\[ \tan \theta = \ldots \quad \tan \alpha = \ldots \]
Solutions 3

1.  \( \sin \theta = \frac{5.8}{20} = .2900 \)  \( \sin \alpha = \frac{19.1}{20} = .9550 \)

\( \cos \theta = \frac{19.1}{20} = .9550 \)  \( \cos \alpha = \frac{5.8}{20} = .2900 \)

\( \tan \theta = \frac{5.8}{19.1} = .3037 \)  \( \tan \alpha = \frac{19.1}{5.8} = 3.2931 \)

\( \theta = 17^\circ \)  \( \alpha = 73^\circ \)

2.  \( \sin \theta = \frac{23.4}{38} = .6158 \)  \( \sin \alpha = \frac{30}{38} = .7895 \)

\( \cos \theta = \frac{30}{38} = .7895 \)  \( \cos \alpha = \frac{23.4}{38} = .6158 \)

\( \tan \theta = \frac{23.4}{30} = .7800 \)  \( \tan \alpha = \frac{30}{23.4} = 1.2821 \)

\( \theta = 38^\circ \)  \( \alpha = 52^\circ \)

3.  \( \sin \theta = \frac{27}{30} = .9000 \)  \( \sin \alpha = \frac{13}{30} = .4333 \)

\( \cos \theta = \frac{13}{30} = .4333 \)  \( \cos \alpha = \frac{27}{30} = .9000 \)

\( \tan \theta = \frac{27}{13} = 2.0769 \)  \( \tan \alpha = \frac{13}{27} = .4815 \)

\( \theta = 64^\circ \)  \( \alpha = 26^\circ \)

4.  \( \sin \theta = \frac{9.2}{19} = .4842 \)  \( \sin \alpha = \frac{16.6}{19} = .8737 \)

\( \cos \theta = \frac{16.6}{19} = .8737 \)  \( \cos \alpha = \frac{9.2}{19} = .4842 \)

\( \tan \theta = \frac{9.2}{16.6} = .5542 \)  \( \tan \alpha = \frac{16.6}{9.2} = 1.8043 \)

\( \theta = 29^\circ \)  \( \alpha = 61^\circ \)
Use the Trig Table to Solve for the Unknown

In lesson 3, we learned how to find the measure of an angle when given the length of the two sides. In this lesson, we are going to learn how to find the length of a side when given the angle and the length of another side. Let’s start with an easier problem and then move to more difficult problems. In a 30°– 60°– 90° triangle, we know the ratio of the short side to the hypotenuse is 1 to 2 or 1/2 or .5000. When the short side is 8, we know the hypotenuse is 16.

Example 1
Find H.

The equation looks like this:

\[
\sin 30^\circ = \frac{8}{H}
\]

Multiply both sides by H.

\[H \sin 30^\circ = 8\]

Divide both sides by \sin 30^\circ.

\[H = \frac{8}{\sin 30^\circ}\]

Replace \sin 30^\circ with the ratio, which is .5000.

\[H = \frac{8}{.5000}\]

Solve for H.

\[H = 16\]
We knew two pieces of information, so we were able to solve for the missing information, in this case the length of the hypotenuse.

**Example 2**

Find \( A \).

\[
\sin 40^\circ = \frac{A}{6}
\]

Multiply both sides by 6.

\[
6 \sin 40^\circ = A
\]

Replace \( \sin 40^\circ \) with .6428.

\[
(6)(.6428) = A
\]

Multiply.

\[
3.8568 = A
\]

We choose to round to hundredths.

\[
3.86 = A
\]

It was natural to use the sine to solve for the missing side since we were given an angle, the side opposite the angle, and the hypotenuse. We could also have solved this problem by using the cosine. The missing angle must be 50°, since the angles are complementary. So \( \cos 50^\circ = A/6 \), and we can continue the process.

\[
\cos 50^\circ = \frac{A}{6}
\]

Multiply both sides by 6.

\[
6 \cos 50^\circ = A
\]

Replace \( \cos 50^\circ \) with .6428.

\[
(6)(.6428) = A
\]

Multiply and round to hundredths.

\[
3.86 = A
\]

In real life, the context will determine how many places to use for your answer.
Example 3
Find B.

\[
\begin{align*}
\tan 29^\circ &= \frac{B}{4} \\
B \tan 29^\circ &= 4 \\
B &= \frac{4}{\tan 29^\circ} \\
B &= \frac{4}{0.5543} \\
B &= 7.22
\end{align*}
\]

In this example, we choose to use the tangent to solve the problem efficiently, since we are interested in the sides opposite and adjacent to the angle

\[
\tan 29^\circ = \frac{B}{4}
\]

Multiply both sides by B.

\[B \tan 29^\circ = 4\]  Divide both sides by \tan 29^\circ.

\[B = \frac{4}{\tan 29^\circ}\]  Replace \tan 29^\circ with the ratio, which is 0.5543.

\[B = \frac{4}{0.5543}\]  Divide and round to hundredths.

\[B = 7.22\]

We could have found the other angle by subtracting \((90^\circ - 29^\circ = 61^\circ)\), and used the equation \(\tan 61^\circ = \frac{B}{4}\) to find B. The equation would have been easier than the one we used in example 3, since no division is involved.

Example 4

\[
\begin{align*}
\tan 61^\circ &= \frac{B}{4} \\
4 \tan 61^\circ &= B \\
(4)(1.804) &= B
\end{align*}
\]

Multiply both sides by 4.

\[4 \tan 61^\circ = B\]  Replace \tan 61^\circ with 1.804.

\[(4)(1.804) = B\]  Multiply and round to hundredths.

\[7.22 = B\]
Practice Problems 1
To solve for the unknown, carefully examine what information is given, and choose the trig function that will solve the problem most efficiently. Remember that there is more than one way to solve each problem. Estimate your answer.

1. 

2. 

3. 

4. 

5. 

6.
Solutions 1

1. \[ \sin 47^\circ = \frac{A}{5} \]
   \[ A = 5 \sin 47^\circ \]
   \[ A = 3.66 \]

2. \[ \cos 33^\circ = \frac{B}{8.2} \]
   \[ B = 8.2 \cos 33^\circ \]
   \[ B = 6.88 \]

3. \[ \sin 25^\circ = \frac{3}{C} \]
   \[ C \sin 25^\circ = 3 \]
   \[ C = \frac{3}{\sin 25^\circ} \]
   \[ C = 7.10 \]

4. \[ \tan 64^\circ = \frac{D}{14} \]
   \[ D = 14 \tan 64^\circ \]
   \[ D = 28.70 \]

5. \[ \cos 9^\circ = \frac{.9}{E} \]
   \[ E = \frac{.9}{\cos 9^\circ} \]
   \[ E = .91 \]

6. \[ \tan 38^\circ = \frac{F}{23} \]
   \[ F = 23 \tan 38^\circ \]
   \[ F = 17.97 \]
I wanted you to use the trig table so you would understand that it is a compilation of rational numbers, or ratios, written as decimals. But it is easier, quicker, and more accurate to use a scientific calculator that has the trigonometric functions. Here is how to select and use one. (This is how they usually operate. Consult the operating instructions that came with your calculator to be sure.)

If you have a calculator with keys for \( \sin \), \( \cos \), \( \tan \), then you are all set. Usually you will also have a button that says INV for inverse or buttons that enable you to get to functions labeled \( \sin^{-1} \), \( \cos^{-1} \), or \( \tan^{-1} \). Hopefully yours also has a \( \frac{1}{x} \) key. If not, when you go shopping, pick one up with these characteristics. Generally, if it has these capabilities, it will also be able to solve for squares and square roots and find logarithms.

Later, we will be studying graphing. While a graphing calculator isn’t necessary for our study of trigonometry, one is nice if you will be doing more advanced math in the future.

There is also a key that switches the value of the numbers on the screen from “degree” to “grad” to “radian” (or rad). At this point in our study you'll want to have “degree” showing.

Now here’s how to use your calculator. Compare your results with the trig table in lesson 3 to make sure you are on the right track. To find a trig ratio with a calculator instead of the table, punch in the number of degrees, hit the trig function, and your decimal ratio will be shown.
Example 1
Find the decimal ratio for sin 30°.

Enter 30 into your calculator and then hit $\sin$.

The screen should read .5.

Example 2
Find the decimal ratio for tan 81°.

Enter 81 into your calculator and then hit $\tan$.

The screen should read $6.313751$, or 6.3138 when rounded to four places or ten-thousandths as in the table.

Practice Problems 1
Find the decimal ratio for each angle. Use your calculator and round to ten-thousandths.

1. sin 14º  
2. tan 88º  
3. tan 67º  
4. cos 36º  
5. sin 52º  
6. cos 45º

Solutions 1

1. sin 14º = .2419  
2. tan 88º = 28.6363  
3. tan 67º = 2.3559  
4. cos 36º = .8090  
5. sin 52º = .7880  
6. cos 45º = .7071
ARC FUNCTIONS

The third part of this lesson tells us how to use the calculator to find degrees when we are given the decimal ratio. We will also learn a new term to indicate this operation. We read \( \sin 30^\circ = \frac{1}{2} \) as “the sin of 30° is one-half.” To go in the opposite direction, we say, “The angle whose sine is one-half is 30°.” This expression is written as: \( \arcsin \frac{1}{2} = 30^\circ \), so \( \text{arc} \) stands for “the angle whose.” Using a decimal, the expression is \( \arcsin (.5000) = 30^\circ \).

To summarize:

- Given the angle, find the ratio.
  
  For sine \( 30^\circ = .5000 \), say, “The sine of 30° is .5000.”

- Given the ratio, find the angle.
  
  For \( \arcsin (.5000) = 30^\circ \), say, “The angle whose sine is .5000 is 30°.”

  For \( \arccos (.5000) = 60^\circ \), say, “The angle whose cosine is .5000 is 60°.”

  For \( \arctan (1.000) = 45^\circ \), say, “The angle whose tangent is 1.000 is 45°.”

The arcsin is referred to as the **inverse function**. \( \sin^{-1} \) is the inverse of \( \sin \). When given the ratio, we use the arc, or inverse, in order to find the angle measure. In lesson 3 we divided two numbers to find the ratio, and then looked at the table to find the nearest angle. Now, instead of looking at the table, we can use a calculator to find the angle.

To use the calculator, first find out whether there is an \( \text{arcsin} \) button and an \( \text{INV} \) button, or if you need to push \( 2\text{ND} \) \( \text{sin} \) to get \( \sin^{-1} \), which is above the \( \text{sin} \) button.

If you are given \( \sin .5000 \) and want to find the measure of the angle, enter .5 and do one of the following:

\[
\begin{align*}
2\text{ND} \text{sin} \quad &\text{or} \quad \text{INV} \text{sin} \quad \text{or} \quad \text{arc} \text{sin} \quad \text{to get} \quad \sin^{-1}.
\end{align*}
\]

Work with your calculator until you understand how to find the ratio when given the angle and how to find the angle when given the ratio.
Practice Problems 2
Use a calculator to find the degrees rounded to the hundredths place.

1. \( \arcsin (.8387) = \)
2. \( \arccos (.3855) = \)
3. \( \arccos (.0954) = \)
4. \( \arccos (.6378) = \)
5. \( \arctan (3.5971) = \)
6. \( \arcsin (.9874) = \)

Solutions 2
1. \( \arcsin (.8387) = 57.00^\circ \)
2. \( \arccos (.3855) = 67.33^\circ \)
3. \( \arccos (.0954) = 84.53^\circ \)
4. \( \arccos (.6378) = 50.37^\circ \)
5. \( \arctan (3.5971) = 74.46^\circ \)
6. \( \arcsin (.9874) = 80.89^\circ \)

DEGREES - MINUTES - SECONDS

Degrees are often given in a form other than decimals. For more accuracy, they are broken down into minutes and seconds. The expression \( 34^\circ 20' 45'' \) is read as “thirty-four degrees, twenty minutes, and forty-five seconds.” To change this to a decimal so that we can use the calculator, remember that there are 60 seconds in a minute and 60 minutes in a degree.

Example 3
Change \( 34^\circ 20' 45'' \) to a decimal, using unit multipliers.

Change the seconds to minutes first:

\[
\frac{45 \text{ seconds}}{1} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{45 \text{ minutes}}{60} = .75 \text{ minutes}
\]
Change the minutes to degrees:

\[
\frac{20.75 \text{ min}}{1 \text{ min}} \times \frac{1 \text{ deg}}{60 \text{ min}} = \frac{20.75 \text{ deg}}{60} = 0.3458 \text{ degrees}
\]
(rounds to 0.35º)

So 34º 20' 45" = 34.35º.

Now we can find the trig ratio by entering 34.35º and using sine, cosine, or tangent to find the ratio.

Practice Problems 3
Change the degrees-minutes-seconds to a decimal number using unit multipliers.

1. 28º 30' 25"
2. 53º 42' 10"
3. 36º 24'
4. 41º 15' 48"
5. 18º 50' 08"
6. 62º 00' 06"

Solutions 3

1. 28.51º
2. 53.70º
3. 36.4º
4. 41.26º
5. 18.84º
6. 62.00º
Now we will learn how to change from degrees–minutes–seconds to a decimal number by using a calculator. This procedure may vary from one calculator to another. DMS to DD is the key button. DD stands for decimal degrees, and DMS is degrees-minutes-seconds. If your calculator does not have these buttons, refer to your owner’s manual.

**Example 4**

Enter 28º 30’ 25” as 28.3025.

Push the button $\text{DMS\rightarrow DD}$.

It shows 28.50694 or 28.51º (rounded).

Do #2–5 from practice problems 3 again to check your work.

Now change from a decimal to degrees-minutes-seconds. DD to DMS is the key button.

**Example 5**

Enter 28.50694 (DD).

Push the button $\text{DD\rightarrow DMS}$.

It shows 28.30249, representing 28º 30’ 25” (DMS).

Change your answers to #2–5 from practice problems 3 back to DMS in order to check your work.

Now that we can use a calculator to change back and forth from DMS to DD, let’s take it another step and find a trig ratio.
Example 6
Find the decimal ratio for sin $15^\circ 24' 50"$.

Enter 15.2450 (DMS).

Push the button $\text{DMS}\rightarrow\text{DD}$, it shows 15.413888 (DD).

Push the button $\sin$.

It reads .2657898, which may be rounded to .2658.

Practice Problems 4
Use the calculator to find the decimal ratio for each angle.

1. cos $7^\circ 34'$
2. sin $68^\circ 27' 43"$
3. sin $36^\circ 48'$
4. tan $24^\circ 12' 12"$
5. tan $50^\circ 08' 22"$
6. cos $47^\circ 00' 05"$

Solutions 4

1. cos $7^\circ 34' = .9913$
2. sin $68^\circ 27' 43" = .9302$
3. sin $36^\circ 48' = .5990$
4. tan $24^\circ 12' 12" = .4495$
5. tan $50^\circ 08' 22" = 1.1977$
6. cos $47^\circ 00' 05" = .6820$
Example 7
What is arcsin (.5222)?

\[
\text{arcsin (.5222)} = 31.48^\circ \text{ or } 31^\circ \ 28' \ 48''
\]

When estimating this kind of problem, think of .48 as a little less than half of a degree. A little less than one-half of 60 minutes would be 28 or 29. So 28' 48'' (28 minutes 48 seconds) is what we should expect.

Practice Problems 5
1. \(\text{arctan (.6565)} = \ldots^\circ\)
2. \(\text{arcsin (.0911)} = \ldots^\circ\)
3. \(\text{arcsin (.8874)} = \ldots^\circ\)
4. \(\text{arccos (.9600)} = \ldots^\circ\)
5. \(\text{arccos (.3131)} = \ldots^\circ\)
6. \(\text{arctan (1.5988)} = \ldots^\circ\)

Solutions 5
1. \(\text{arctan (.6565)} = 33.28^\circ \text{ or } 33^\circ \ 17' \ 06''\)
2. \(\text{arcsin (.0911)} = 5.23^\circ \text{ or } 5^\circ \ 13' \ 37''\)
3. \(\text{arcsin (.8874)} = 62.55^\circ \text{ or } 62^\circ \ 32' \ 54''\)
4. \(\text{arccos (.9600)} = 16.26^\circ \text{ or } 16^\circ \ 15' \ 37''\)
5. \(\text{arccos (.3131)} = 71.75^\circ \text{ or } 71^\circ \ 45' \ 14''\)
6. \(\text{arctan (1.5988)} = 57.98^\circ \text{ or } 57^\circ \ 58' \ 31''\)
Example 8

\[ \cos 17.3^\circ = \frac{8}{B} \]

\[ B = \frac{8}{\cos 17.3^\circ} = \frac{8}{0.9548} \]

\[ B = 8.38 \]

Practice Problems 6
Use your calculator to solve for the missing side or angle.

1. \[ \angle 27^\circ \quad 12 \]

2. \[ \theta \quad 29.4 \quad 17.5 \]

3. \[ \angle 39.2^\circ \quad 5.7 \]

4. \[ \theta \quad 12 \quad 9.7 \]

5. \[ \angle 38^\circ \quad 10.4 \]

6. \[ \angle 44.1^\circ \quad 0.87 \]
### Solutions 6

1. \( \tan 27^\circ = \frac{A}{12} \)  
   \((12) \tan 27^\circ = A \)  
   \((12)(.5095) = A \)  
   \( 6.11 = A \)

2. \( \tan \theta = \frac{17.5}{29.4} \)  
   \( \frac{17.5}{29.4} = .5952 \)  
   \( \theta = 30.76^\circ \)

3. \( \cos 39.2^\circ = \frac{5.7}{B} \)  
   \( B = \frac{5.7}{\cos 39.2^\circ} = \frac{5.7}{.7749} \)  
   \( B = 7.36 \)

4. \( \tan \theta = \frac{12}{9.7} \)  
   \( \frac{12}{9.7} = 1.2371 \)  
   \( \theta = 51.05^\circ \)

5. \( \cos 38^\circ = \frac{10.4}{C} \)  
   \( C = \frac{10.4}{\cos 38^\circ} = \frac{10.4}{.7880} \)  
   \( C = 13.20 \)

6. \( \sin 44.1^\circ = \frac{D}{.87} \)  
   \( (.87) \sin 44.1^\circ = D \)  
   \( .61 = D \)
Pythagorean Theorem

This is a right triangle. There are three sides. The two sides that join to form the right angle are called the legs. You can remember this by the letter $L$ for Leg, because $L$ makes a right angle. Besides the two legs, there is the longest side, which is called the hypotenuse. (Hypotenuse is the longest word and the longest side!)

The most familiar right triangle is the 3–4–5 right triangle. Here is a picture of this triangle, showing the Pythagorean theorem, which is “leg squared plus leg squared equals hypotenuse squared.”
If you have a right triangle, then the Pythagorean theorem works. The converse is also true. If leg squared plus leg squared equals hypotenuse squared, then the triangle is a right triangle. Look at these examples to make sure you understand this important theorem.

**Example 1**

\[ \text{Leg}^2 + \text{Leg}^2 = \text{Hyp}^2 \]

\[ 6^2 + 7^2 = H^2 \]
\[ 36 + 49 = H^2 \]
\[ 85 = H^2 \]
\[ \sqrt{85} = H \]

**Example 2**

A triangle has sides of 6, 8, and 10. Is it a right triangle?

\[ 6^2 + 8^2 = 10^2 \]
\[ 36 + 64 = 100 \]
Yes, it is a right triangle.

**Example 3**

\[ \text{Leg}^2 + \text{Leg}^2 = \text{Hyp}^2 \]

\[ 3^2 + L^2 = 8^2 \]
\[ 9 + L^2 = 64 \]
\[ L^2 = 55 \]
\[ L = \sqrt{55} \]

**Example 4**

A triangle has sides of 4, 5, and 6. Is it a right triangle?

\[ 4^2 + 5^2 = 6^2 \]
\[ 16 + 25 = 36 \]
No, it is not a right triangle.
Special Triangles (45°-45°-90°; 30°-60°-90°)

Have you noticed the relationship between the legs and the hypotenuse of a 45°-45°-90° right triangle? Since the angles are congruent, the opposite sides are also congruent. Think of the triangle shown as half of a square.

\[ 45° \quad 45° \]

Find the length of the hypotenuse of the following triangles, and observe the common thread.

\[ 6^2 + 6^2 = H^2 \]
\[ 36 + 36 = H^2 \]
\[ 72 = H^2 \]
\[ \sqrt{72} = H \]
\[ \sqrt{36 \cdot 2} = H \]
\[ 6\sqrt{2} = H \]

\[ 10^2 + 10^2 = H^2 \]
\[ 100 + 100 = H^2 \]
\[ 200 = H^2 \]
\[ \sqrt{200} = H \]
\[ \sqrt{100 \cdot 2} = H \]
\[ 10\sqrt{2} = H \]
Now let’s try a tricky one with A as the length of the legs.

\[ A^2 + A^2 = H^2 \]
\[ 2A^2 = H^2 \]
\[ \sqrt{2A^2} = \sqrt{H^2} \]
\[ \sqrt{A^2 \cdot \sqrt{2}} = H \]
\[ A\sqrt{2} = H \]

The hypotenuse is \( \sqrt{2} \) times the leg. If the leg is 5, then the hypotenuse will be \( 5\sqrt{2} \). If the leg is \( x \), then the hypotenuse is \( x\sqrt{2} \).

We employed the Pythagorean theorem to discover this unique relationship between the legs and the hypotenuse in a 45°-45°-90° right triangle. Notice that this is true regardless of the lengths of the legs or of the hypotenuse. For any 45°-45°-90° triangle, whatever the length of one leg is, the other leg will be the same length, and the hypotenuse will be \( \sqrt{2} \) times that length!

Not only does the Pythagorean theorem reveal this, but it also proves it to be so for any 45°-45°-90° right triangle. The ratio between the legs, and between a leg and the hypotenuse, is constant, i.e. always true.

Another special right triangle is the 30°-60°-90° right triangle. Let’s do a couple of examples to observe the relationship and discover the pattern.

\[ 4^2 + \left(4\sqrt{3}\right)^2 = H^2 \]
\[ 16 + 16 \cdot 3 = H^2 \]
\[ 16 + 48 = H^2 \]
\[ \sqrt{64} = H^2 \]
\[ 8 = H \]

\[ 7^2 + \left(7\sqrt{3}\right)^2 = H^2 \]
\[ 49 + 49 \cdot 3 = H^2 \]
\[ 49 + 147 = H^2 \]
\[ 196 = H^2 \]
\[ 14 = H \]
Now predict the hypotenuse in this triangle (based on your observations); and then work it out to confirm your hypothesis.

\[
S^2 + \left(\sqrt{3}\right)^2 = H^2
\]

The hypotenuse is twice the length of the short side (the side opposite the smallest angle). If you are given the short leg, you double it to find the hypotenuse.

Now let’s find the relationship between the short leg and the long leg.

We double the short leg to find the hypotenuse, which is 10. Our equation to find the long leg is:

\[
5^2 + (LL)^2 = 10^2
\]

\[
25 + (LL)^2 = 100
\]

\[
(\sqrt{25})^2 = 75
\]

\[
LL = \sqrt{25\sqrt{3}}
\]

\[
LL = 5\sqrt{3}
\]

Now let’s use variables to confirm our guess. If the hypotenuse is 2B, then the short leg is B.

\[
(B)^2 + (LL)^2 = (2B)^2
\]

\[
B^2 + (LL)^2 = 4B^2
\]

\[
(\sqrt{\frac{3}{2}})^2 = 3B^2
\]

\[
LL = \sqrt{B\sqrt{3}}
\]

\[
LL = B\sqrt{3}
\]

The long leg is \(\sqrt{3}\) times the short leg.
In the 30°-60°-90° right triangle, the relationship between the sides is not determined by the lengths of the sides, but by the size or measure of the angles. The side opposite the 30° angle is always one-half the length of the hypotenuse (the side opposite the right angle). The side opposite the 60° angle is always $\sqrt{3}$ times the side opposite the 30° angle. These ratios are “set in concrete” so to speak—that is, they are always true.
Lesson 1A

1. $\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
2. $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
3. $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{5} = 1$
4. $\sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{5}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
5. $\cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
6. $\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{5}{5} = 1$
7. $\sin \theta = \frac{13}{26} = \frac{1}{2}$
8. $\cos \theta = \frac{13\sqrt{3}}{26} = \sqrt{3}$
9. $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{13}{13} = \frac{1}{\sqrt{3}}$
10. $\sin \alpha = \frac{13\sqrt{3}}{26} = \frac{\sqrt{3}}{2}$
11. $\cos \alpha = \frac{13}{26} = \frac{1}{2}$
12. $\tan \alpha = \frac{13\sqrt{3}}{13} = \sqrt{3}$
13. $\sin \theta = \frac{7}{\sqrt{130}} = \frac{7\sqrt{130}}{130}$
14. $\cos \theta = \frac{9}{\sqrt{130}} = \frac{9\sqrt{130}}{130}$
15. $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{9}$
16. $\sin \alpha = \frac{9}{\sqrt{130}} = \frac{9\sqrt{130}}{130}$
17. $\cos \alpha = \frac{7}{\sqrt{130}} = \frac{7\sqrt{130}}{130}$

Lesson 1B

18. $\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{9}{7}$
19. $\sin \theta = \frac{\sqrt{203}}{18}$
20. $\cos \theta = \frac{11}{18}$
21. $\tan \theta = \frac{\sqrt{203}}{11}$
22. $\sin \alpha = \frac{11}{18}$
23. $\cos \alpha = \frac{\sqrt{203}}{18}$
24. $\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{11}{\sqrt{203}} = \frac{\sqrt{203}}{203}$
Lesson 1B

13. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3.5}{12.5} = \frac{7}{25} \)

14. \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{12.5} = \frac{24}{25} \)

15. \( \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{3.5}{12} = \frac{7}{24} \)

16. \( \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{12}{12.5} = \frac{24}{25} \)

17. \( \cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{3.5}{12.5} = \frac{7}{25} \)

18. \( \tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{12}{3.5} = \frac{24}{7} \)

19. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{9} \)

20. \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{8.5}{9} = \frac{17}{18} \)

21. \( \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{8.5} = \frac{8}{17} \)

22. \( \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{8.5}{9} = \frac{17}{18} \)

23. \( \cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{4}{9} \)

24. \( \tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{8.5}{4} = \frac{17}{8} \)

Lesson 1C

1. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{12.8} = \frac{80}{128} = \frac{5}{8} \)

2. \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{10}{12.8} = \frac{5}{6.4} = \frac{50}{64} = \frac{25}{32} \)

3. \( \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{10} = \frac{4}{5} \)

4. \( \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{10}{12.8} = \frac{5}{6.4} = \frac{50}{64} = \frac{25}{32} \)

5. \( \cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{8}{12.8} = \frac{80}{128} = \frac{5}{8} \)

6. \( \tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{10}{8} = \frac{5}{4} \)

7. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{12}{13.4} = \frac{120}{134} = \frac{60}{67} \)

8. \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{13.4} = \frac{60}{134} = \frac{30}{67} \)

9. \( \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{12}{6} = 2 \)

10. \( \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{6}{13.4} = \frac{60}{134} = \frac{30}{67} \)

11. \( \cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13.4} = \frac{120}{134} = \frac{60}{67} \)

12. \( \tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{6}{12} = \frac{1}{2} \)

13. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{13.6}{15} = \frac{136}{150} = \frac{68}{75} \)

14. \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{6.4}{15} = \frac{64}{150} = \frac{32}{75} \)

15. \( \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{13.6}{6.4} = \frac{68}{32} = \frac{17}{8} \)

16. \( \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{6.4}{15} = \frac{64}{150} = \frac{32}{75} \)

17. \( \cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{13.6}{15} = \frac{136}{150} = \frac{68}{75} \)

18. \( \tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{6.4}{13.6} = \frac{64}{136} = \frac{8}{17} \)

19. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{X}{2X} = \frac{1}{2} \)

20. \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{X\sqrt{3}}{2X} = \frac{\sqrt{3}}{2} \)

21. \( \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{X}{X\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \)

22. \( \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{X\sqrt{3}}{2X} = \frac{\sqrt{3}}{2} \)

23. \( \cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{X}{2X} = \frac{1}{2} \)

24. \( \tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{X\sqrt{3}}{X} = \sqrt{3} \)

Lesson 2A

1. \( 6^2 + 9^2 = H^2 \)

\( 36 + 81 = H^2 \)

\( 117 = H^2 \)

\( H \approx 10.82 \)

\( \sin \theta = \frac{6}{10.82} \approx 0.5545 \)

\( \cos \theta = \frac{9}{10.82} \approx 0.8318 \)

\( \tan \theta = \frac{6}{9} = 0.6667 \)

\( \csc \theta = \frac{10.82}{6} \approx 1.7833 \)

\( \sec \theta = \frac{10.82}{9} \approx 1.2022 \)

\( \cot \theta = \frac{9}{6} \approx 1.5000 \)
2. \[ 13^2 + 7^2 = H^2 \]
\[ 196 + 49 = H^2 \]
\[ 245 = H^2 \]
\[ H = \sqrt{245} \approx 15.65 \]

\[ \sin \theta = \frac{7}{15} \approx .4743 \]
\[ \cos \theta = \frac{13}{15} \approx .8680 \]
\[ \tan \theta = \frac{7}{13} \approx .5385 \]
\[ \csc \theta = \frac{15}{7} \approx 2.1571 \]
\[ \sec \theta = \frac{13}{7} \approx 1.8571 \]
\[ \cot \theta = \frac{13}{7} \approx .8571 \]

3. \[ 7^2 + L^2 = 9.9^2 \]
\[ 49 + L^2 = 98.01 \]
\[ L^2 = 98.01 - 49 \]
\[ L^2 = 49.01 \]
\[ L = \sqrt{49.01} \approx 7 \]

\[ \sin \theta = \frac{7}{9.9} \approx .7071 \]
\[ \cos \theta = \frac{9.9}{9.9} \approx 1 \]
\[ \tan \theta = \frac{7}{9.9} \approx 1.0000 \]
\[ \csc \theta = \frac{9.9}{7} \approx 1.4143 \]
\[ \sec \theta = \frac{9.9}{7} \approx 1.4143 \]
\[ \cot \theta = \frac{7}{9.9} \approx 1.0000 \]

4. \[ 8^2 + L^2 = 15^2 \]
\[ 64 + L^2 = 225 \]
\[ L^2 = 225 - 64 \]
\[ L^2 = 161 \]
\[ L = \sqrt{161} \approx 12.69 \]

\[ \sin \theta = \frac{12.69}{15} \approx .8460 \]
\[ \cos \theta = \frac{8}{15} \approx .5333 \]
\[ \tan \theta = \frac{12.69}{8} \approx 1.5863 \]
\[ \csc \theta = \frac{15}{12.69} \approx 1.1820 \]
\[ \sec \theta = \frac{15}{8} \approx 1.8750 \]
\[ \cot \theta = \frac{8}{12.69} \approx .6304 \]

Lesson 2B

1. \[ 4.5^2 + 6.6^2 = H^2 \]
\[ 20.25 + 43.56 = H^2 \]
\[ 63.81 = H^2 \]
\[ H = \sqrt{63.81} \approx 7.99 \]

\[ \sin \theta = \frac{4.5}{7.99} \approx .5632 \]
\[ \cos \theta = \frac{6.6}{7.99} \approx .8260 \]
\[ \tan \theta = \frac{4.5}{6.6} \approx .6818 \]
\[ \csc \theta = \frac{7.99}{4.5} \approx 1.7756 \]
\[ \sec \theta = \frac{7.99}{6.6} \approx 1.2106 \]
\[ \cot \theta = \frac{6.6}{4.5} \approx 1.4667 \]
Lesson 2B - Lesson 2C

2. 
\[L^2 + 6.8^2 = \sqrt{51}^2\]
\[L^2 + 46.24 = 51\]
\[L^2 = 51 - 46.24\]
\[L = \sqrt{4.76}\]
\[L \approx 2.18\]

5. 
\[(2\sqrt{3})^2 + 11^2 = H^2\]
\[2\sqrt{3} \approx 3.46\]
\[(2)(2\sqrt{3}/3 + 121 = H^2\]
\[4(3) + 121 = H^2\]
\[133 = H^2\]
\[H \approx 11.53\]

3. 
\[10^2 + \sqrt{3}^2 = H^2\]
\[\sqrt{3} \approx 1.73\]
\[100 + 3 = H^2\]
\[103 = H^2\]
\[\sqrt{103} = H\]
\[H \approx 10.15\]

6. 
\[5^2 + 12^2 = H^2\]
\[25 + 144 = H^2\]
\[169 = H^2\]
\[H \approx 13\]

Lesson 2C

4. 
\[(\sqrt{7})^2 + (\sqrt{7})^2 = H^2\]
\[7\sqrt{2} \approx 9.9\]
\[7\sqrt{3} \approx 12.12\]
\[98 + 147 = H^2\]
\[245 = H^2\]
\[H = 15.65\]

1. 
\[9^2 + 8^2 = H^2\]
\[7\sqrt{2} \approx 9.9\]
\[81 + 64 = H^2\]
\[145 = H^2\]
\[\sqrt{145} = H\]
\[H \approx 12.04\]
2. $5^2 + 5^2 = H^2$
   $25 + 25 = H^2$
   $50 = H^2$
   $\sqrt{50} = H$
   $H \approx 7.07$
   $\sin \theta = \frac{5}{7.07} \approx .7072$
   $\cos \theta = \frac{5}{7.07} \approx .7072$
   $\tan \theta = \frac{5}{5} = 1.0000$
   $\csc \theta = \frac{7.07}{5} = 1.4140$
   $\sec \theta = \frac{7.07}{5} = 1.4140$
   $\cot \theta = \frac{5}{5} = 1.0000$

3. $3^2 + 3^2 = H^2$
   $9 + 9 = H^2$
   $18 = H^2$
   $\sqrt{18} = H$
   $H = 4.24$
   $\sin \theta = \frac{3}{4.24} \approx .7075$
   $\cos \theta = \frac{3}{4.24} \approx .7075$
   $\tan \theta = \frac{3}{3} = 1.0000$
   $\csc \theta = \frac{4.24}{3} \approx 1.4133$
   $\sec \theta = \frac{4.24}{3} \approx 1.4133$
   $\cot \theta = \frac{3}{3} = 1.0000$

4. $12^2 + L^2 = 13^2$
   $144 + L^2 = 169$
   $L^2 = 169 - 144$
   $L^2 = 25$
   $L = \sqrt{25}$
   $L = 5$
   $\sin \theta = \frac{5}{13} \approx .3846$
   $\cos \theta = \frac{12}{13} \approx .9231$
   $\tan \theta = \frac{5}{12} \approx .4167$
   $\csc \theta = \frac{13}{5} = 2.6000$
   $\sec \theta = \frac{13}{5} \approx 1.0833$
   $\cot \theta = \frac{12}{5} = 2.4000$

5. $4^2 + 6^2 = H^2$
   $16 + 36 = H^2$
   $52 = H^2$
   $\sqrt{52} = H$
   $H \approx 7.21$
   $\sin \theta = \frac{6}{7.21} \approx .8322$
   $\cos \theta = \frac{4}{7.21} \approx .5548$
   $\tan \theta = \frac{6}{4} = 1.5000$
   $\csc \theta = \frac{7.21}{6} \approx 1.2017$
   $\sec \theta = \frac{7.21}{4} \approx 1.8025$
   $\cot \theta = \frac{4}{6} \approx .6667$

6. $7^2 + 7^2 = H^2$
   $49 + 49 = H^2$
   $98 = H^2$
   $\sqrt{98} = H$
   $H \approx 9.90$
   $\sin \theta = \frac{7}{9.9} \approx .7071$
   $\cos \theta = \frac{7}{9.9} \approx .7071$
   $\tan \theta = \frac{7}{7} = 1.0000$
   $\csc \theta = \frac{9.9}{7} \approx 1.4143$
   $\sec \theta = \frac{9.9}{7} \approx 1.4143$
   $\cot \theta = \frac{7}{7} = 1.0000$

Lesson 3A
1. $\cos 37^\circ = .7986$
2. $\tan 51^\circ = 1.2349$
3. $\sin 20^\circ = .3420$
4. $\sin 49^\circ = .7547$
5. $\cos 65^\circ = .4226$
6. $.6249 = \tan 32^\circ$
7. $.4540 = \cos 63^\circ$
8. $.0875 = \tan 5^\circ$
9. $.9781 = \sin 78^\circ$
10. $14.3007 = \tan 86^\circ$
11. $\sin \theta = \frac{20}{26.9} \approx .7435$
    $\cos \theta = \frac{18}{26.9} \approx .6691$
    $\tan \theta = \frac{20}{18} \approx 1.1111$
    $\theta = 48^\circ$
    $\sin \alpha = \frac{18}{26.9} \approx .6691$
    $\cos \alpha = \frac{20}{26.9} \approx .7435$
    $\tan \alpha = \frac{18}{20} = .9000$
    $\alpha = 42^\circ$
Lesson 3A
1. \( \sin 12^\circ = .2079 \)
2. \( \cos 86^\circ = .0698 \)
3. \( \sin 40^\circ = .6428 \)
4. \( \tan 22^\circ = .4040 \)
5. \( \cos 18^\circ = .9511 \)
6. \( .9336 = \sin 69^\circ \)
7. \( 4.0108 = \tan 76^\circ \)
8. \( .9986 = \sin 87^\circ \)
9. \( 1.4826 = \tan 56^\circ \)
10. \( .9925 = \cos 7^\circ \)

Lesson 3B
1. \( \sin \theta = \frac{7}{12.2} \approx .5738 \)
2. \( \cos \theta = \frac{10}{12.2} \approx .8197 \)
3. \( \tan \theta = \frac{7}{10} \approx .7000 \)
4. \( \theta = 35^\circ \)
5. \( \sin \alpha = \frac{10}{12.2} \approx .8197 \)
6. \( \cos \alpha = \frac{7}{12.2} \approx .5738 \)
7. \( \tan \alpha = \frac{10}{7} \approx 1.4286 \)
8. \( \alpha = 55^\circ \)

11. \( \sin \theta = \frac{4.85}{5} \approx .9700 \)
12. \( \cos \theta = \frac{1.2}{5} \approx .2400 \)
13. \( \tan \theta = \frac{4.85}{1.2} \approx 4.0417 \)
14. \( \theta = 76^\circ \)
15. \( \sin \alpha = \frac{1.2}{5} \approx .2400 \)
16. \( \cos \alpha = \frac{4.85}{5} \approx .9700 \)
17. \( \tan \alpha = \frac{1.2}{4.85} \approx 0.2474 \)
18. \( \alpha = 14^\circ \)

12. \( \sin \theta = \frac{14}{19.8} \approx .7071 \)
13. \( \cos \theta = \frac{14}{19.8} \approx .7071 \)
14. \( \tan \theta = \frac{14}{14} = 1.0000 \)
15. \( \theta = 45^\circ \)
16. \( \sin \alpha = \frac{14}{19.8} \approx .7071 \)
17. \( \cos \alpha = \frac{14}{19.8} \approx .7071 \)
18. \( \tan \alpha = \frac{14}{14} = 1.0000 \)
19. \( \alpha = 45^\circ \)

13. \( \sin \theta = \frac{34}{100} \approx .3400 \)
14. \( \cos \theta = \frac{94}{100} \approx .9400 \)
15. \( \tan \theta = \frac{34}{94} \approx .3617 \)
16. \( \theta = 20^\circ \)
17. \( \sin \alpha = \frac{94}{100} \approx .9400 \)
18. \( \cos \alpha = \frac{34}{100} \approx .3400 \)
19. \( \tan \alpha = \frac{94}{34} \approx 2.7647 \)
20. \( \alpha = 70^\circ \)
Lesson 3C

1. \( \sin \theta = \frac{10}{20} = .5000 \)
   \( \cos \theta = \frac{17.3}{20} = .8650 \)
   \( \tan \theta = \frac{10}{17.3} \approx .5780 \)
   \( \theta = 30^\circ \)
   \( \sin \alpha = \frac{17.3}{20} = .8650 \)
   \( \cos \alpha = \frac{10}{20} = .5000 \)
   \( \tan \alpha = \frac{17.3}{10} = 1.7300 \)
   \( \alpha = 60^\circ \)

2. \( \sin \theta = \frac{7}{8} = .8750 \)
   \( \cos \theta = \frac{3.9}{8} = .4875 \)
   \( \tan \theta = \frac{7}{3.9} \approx 1.7949 \)
   \( \theta = 61^\circ \)
   \( \sin \alpha = \frac{3.9}{8} = .4875 \)
   \( \cos \alpha = \frac{7}{8} = .8750 \)
   \( \tan \alpha = \frac{3.9}{7} \approx .5571 \)
   \( \alpha = 29^\circ \)

3. \( (\sqrt{17})^2 + (\sqrt{17})^2 = H^2 \)
   \( \sqrt{17} + \sqrt{17} = H^2 \)
   \( 34 = H^2 \)
   \( \sqrt{34} = H \)
   \( H \approx 5.83 \)
   \( \sin \theta = \frac{4.12}{5.83} \approx .7067 \)
   \( \cos \theta = \frac{4.12}{5.83} \approx .7067 \)
   \( \tan \theta = \frac{4.12}{4.12} = 1.0000 \)
   \( \csc \theta = \frac{5.83}{4.12} \approx 1.4150 \)
   \( \sec \theta = \frac{5.83}{4.12} \approx 1.4150 \)
   \( \cot \theta = \frac{4.12}{4.12} = 1.0000 \)

4. \( 6^2 + 8.4^2 = H^2 \)
   \( \sin \theta = \frac{6}{10.32} \approx .5814 \)
   \( 36 + 70.56 = H^2 \)
   \( 106.56 = H^2 \)
   \( \sqrt{106.56} = H \)
   \( H \approx 10.32 \)
   \( \cos \theta = \frac{8.4}{10.32} \approx .8140 \)
   \( \tan \theta = \frac{6}{8.4} \approx .7143 \)
   \( \csc \theta = \frac{10.32}{6} = 1.7200 \)
   \( \sec \theta = \frac{10.32}{8.4} \approx 1.2286 \)
   \( \cot \theta = \frac{8.4}{6} = 1.4000 \)

5. \( L^2 + 9^2 = 18^2 \)
   \( \sin \theta = \frac{9}{18} = .5000 \)
   \( L^2 + 81 = 324 \)
   \( L^2 = 324 - 81 \)
   \( L^2 = 243 \)
   \( L = \sqrt{243} \)
   \( L = 15.59 \)
   \( \cos \theta = \frac{15.59}{18} \approx .8661 \)
   \( \tan \theta = \frac{9}{15.59} \approx .5773 \)
   \( \csc \theta = \frac{18}{9} = 2.0000 \)
   \( \sec \theta = \frac{18}{15.59} \approx 1.1546 \)
   \( \cot \theta = \frac{15.59}{9} \approx 1.7322 \)

6. \( 5^2 + 6^2 = H^2 \)
   \( \sin \theta = \frac{5}{7.81} \approx .6402 \)
   \( 25 + 36 = H^2 \)
   \( 61 = H^2 \)
   \( \sqrt{61} = H \)
   \( H \approx 7.81 \)
   \( \cos \theta = \frac{6}{7.81} \approx .7682 \)
   \( \tan \theta = \frac{5}{6} \approx .8333 \)
   \( \csc \theta = \frac{7.81}{5} = 1.5620 \)
   \( \sec \theta = \frac{7.81}{6} \approx 1.3017 \)
   \( \cot \theta = \frac{6}{5} = 1.2000 \)
Lesson 3D

1. \( \sin \theta = \frac{12.3}{12.5} \approx 0.9840 \)
   \( \cos \theta = \frac{2.18}{12.5} \approx 0.1744 \)
   \( \tan \theta = \frac{12.3}{2.18} \approx 5.6422 \)
   \( \theta = 80^\circ \)

2. \( \sin \theta = \frac{8}{11} \approx 0.7273 \)
   \( \cos \theta = \frac{7.5}{11} \approx 0.6818 \)
   \( \tan \theta = \frac{8}{7.5} \approx 1.0667 \)
   \( \varphi = 47^\circ \)

3. \( 7^2 + 12^2 = H^2 \)
   \( 49 + 144 = H^2 \)
   \( H = 13.89 \)
   \( \sin \alpha = \frac{7}{12} \approx 0.5833 \)

4. \( 12^2 + 13^2 = H^2 \)
   \( 144 + 169 = H^2 \)
   \( H = 17.69 \)

5. \( 9^2 + 12^2 = H^2 \)
   \( 81 + 144 = H^2 \)
   \( H = 15 \)

6. \( \left(5\sqrt{2}\right)^2 + 14^2 = H^2 \)
   \( 50 + 196 = H^2 \)
   \( H = 15.68 \)

Lesson 4A

1. \( \sin 15^\circ = \frac{7.5}{A} \)
   \( A \sin 15^\circ = 7.5 \)
   \( A = \frac{7.5}{\sin 15^\circ} \approx \frac{7.5}{0.2588} \approx 28.98 \)
LESSON 4A - LESSON 4C

2. \[ \tan 37^\circ = \frac{20}{B} \]
   \[ B \tan 37^\circ = 20 \]
   \[ B = \frac{20}{\tan 37^\circ} \approx \frac{20}{.7536} \approx 26.54 \]

3. \[ \sin 69^\circ = \frac{12}{C} \]
   \[ C \sin 69^\circ = 12 \]
   \[ C = \frac{12}{\sin 69^\circ} \approx \frac{12}{.9336} \approx 12.85 \]

4. \[ \tan 57^\circ = \frac{D}{14} \]
   \[ D = 14 \tan 57^\circ \]
   \[ D \approx 14(1.5399) \approx 21.56 \]

5. \[ \tan 19^\circ = \frac{10}{E} \]
   \[ E \tan 19^\circ = 10 \]
   \[ E = \frac{10}{\tan 19^\circ} \approx \frac{10}{.3443} \approx 29.04 \]

6. \[ \sin 80^\circ = \frac{F}{11.3} \]
   \[ 11.3 \sin 80^\circ = F \]
   \[ F \approx 11.3(.9848) \approx 11.13 \]

Lesson 4B

1. \[ \tan 25^\circ = \frac{G}{15} \]
   \[ 15 \tan 25^\circ = G \]
   \[ G \approx 15(.4663) \approx 6.99 \]

2. \[ \cos 32^\circ = \frac{22}{H} \]
   \[ H \cos 32^\circ = 22 \]
   \[ H = \frac{22}{\cos 32^\circ} \approx \frac{22}{.8480} \approx 25.94 \]

3. \[ \cos 41^\circ = \frac{J}{\sqrt{17}} \]
   \[ \sqrt{17} \cos 41^\circ = J \]
   \[ J \approx \sqrt{17}(.7547) \]
   \[ J \approx (4.12)(.7547) \approx 3.11 \]

4. \[ \tan 62^\circ = \frac{K}{13} \]
   \[ 13 \tan 62^\circ = K \]
   \[ K \approx (13)(1.8807) \approx 24.45 \]

5. \[ \cos 73^\circ = \frac{L}{9.8} \]
   \[ 9.8 \cos 73^\circ = L \]
   \[ L \approx (9.8)(.2924) \approx 2.87 \]

6. \[ \tan 45^\circ = \frac{M}{16} \]
   \[ 16 \tan 45^\circ = M \]
   \[ M \approx (16)(1) = 16 \]

Lesson 4C

1. \[ \sin 21^\circ = \frac{N}{25} \]
   \[ 25 \sin 21^\circ = N \]
   \[ N \approx 25(.3584) \approx 8.96 \]

2. \[ \tan 36^\circ = \frac{1.8}{P} \]
   \[ P \tan 36^\circ = 1.8 \]
   \[ P \approx \frac{1.8}{\tan 36^\circ} \]
   \[ P \approx \frac{1.8}{.7265} \approx 2.48 \]

3. \[ \tan 68^\circ = \frac{Q}{19} \]
   \[ 19 \tan 68^\circ = Q \]
   \[ Q \approx 19(2.4751) \approx 47.03 \]

4. \[ \sin 42^\circ = \frac{R}{\sqrt{32}} \]
   \[ \sqrt{32} \sin 42^\circ = R \]
   \[ R \approx \left(\sqrt{32}\right)(.6691) \]
   \[ R \approx (5.66)(.6691) \approx 3.79 \]

5. \[ \sin 10^\circ = \frac{S}{100} \]
   \[ 100 \sin 10^\circ = S \]
   \[ S \approx 100(.1736) \approx 17.36 \]

6. \[ \tan 54^\circ = \frac{T}{16.1} \]
   \[ 16.1 \tan 54^\circ = T \]
   \[ T \approx 16.1(1.3764) \approx 22.16 \]
Lesson 4D
1. \[ \cos 75^\circ = \frac{30}{U} \]
   \[ \text{Ucos} 75^\circ = 30 \]
   \[ U = \frac{30}{\cos 75^\circ} \]
   \[ U \approx \frac{30}{.2588} \approx 115.92 \]

2. \[ \sin 28^\circ = \frac{\sqrt{9}}{V} \]
   \[ V\sin 28^\circ = \sqrt{9} \]
   \[ V \sin 28^\circ = 3 \]
   \[ V \approx \frac{3}{.4695} \approx 6.39 \]

3. \[ \tan 18^\circ = \frac{7}{W} \]
   \[ W\tan 18^\circ = 7 \]
   \[ W = \frac{7}{\tan 18^\circ} \]
   \[ W \approx \frac{7}{.3249} \approx 21.55 \]

4. \[ \cos 64^\circ = \frac{4.75}{X} \]
   \[ X \cos 64^\circ = 4.75 \]
   \[ X \approx \frac{4.75}{.3834} \approx 10.83 \]

5. \[ \sin 42^\circ = \frac{Y}{22} \]
   \[ 22 \sin 42^\circ = Y \]
   \[ Y \approx 22(0.6691) \approx 14.72 \]

6. \[ \tan 51^\circ = \frac{Z}{18} \]
   \[ 18 \tan 51^\circ = Z \]
   \[ Z \approx 18(1.2349) \approx 22.23 \]

Lesson 5A
1. \[ \tan 28^\circ \approx .5317 \]
2. \[ \sin 33^\circ \approx .5446 \]
3. \[ \cos 40^\circ \approx .7660 \]
4. \[ \tan 5^\circ \approx .0875 \]
5. \[ \cos 72^\circ \approx .3090 \]

6. \[ \sin 69^\circ \approx .9336 \]
7. \[ \arccos .9848 \approx 10^\circ \]
8. \[ \arctan .5317 \approx 28^\circ \]
9. \[ \arcsin .9511 \approx 72^\circ \]
10. \[ \arctan 4.7046 \approx 78^\circ \]
11. \[ \arcsin .4067 \approx 24^\circ \]
12. \[ \arccos .4067 \approx 66^\circ \]
13. \[ 25^\circ 15' 10'' \]
   \[ \frac{10''}{1} \times \frac{1^\circ}{60''} = \left( \frac{10}{60} \right)^\circ \approx .17' \]
   \[ \frac{15.17'}{1} \times \frac{1^\circ}{60'} = \left( \frac{15.17}{60} \right)^\circ \approx .253^\circ \]
   \[ 25^\circ 15' 10'' \approx 25.253^\circ \]
14. \[ 74^\circ 22' 30'' \]
   \[ \frac{30''}{1} \times \frac{1^\circ}{60''} = \left( \frac{30}{60} \right)^\circ = .5' \]
   \[ \frac{22.5'}{1} \times \frac{1^\circ}{60'} = \left( \frac{22.5}{60} \right)^\circ = .375^\circ \]
   \[ 74^\circ 22' 30'' \approx 74.375^\circ \]
15. \[ 32^\circ 48' 12'' \]
   \[ \frac{12''}{1} \times \frac{1^\circ}{60''} = \left( \frac{12}{60} \right)^\circ = .20' \]
   \[ \frac{48.2'}{1} \times \frac{1^\circ}{60'} = \left( \frac{48.2}{60} \right)^\circ \approx .803 \]
   \[ 32^\circ 48' 12'' \approx 32.803^\circ \]
16. \[ 81^\circ 50' \]
   \[ \frac{50'}{1} \times \frac{1^\circ}{60'} = \left( \frac{50}{60} \right)^\circ \approx .833 \]
   \[ 81^\circ 50' \approx 81.833 \]
17. \[ 25^\circ 15' 10'' \approx 25.253^\circ \]
18. \[ 74^\circ 22' 30'' = 74.375^\circ \]
19. \[ 32^\circ 48' 12'' = 32.803^\circ \]
20. \[ 81^\circ 50' = 81.833^\circ \]
21. \[ \arccos .5825 \approx 54.37^\circ \]
22. \[ \arctan 2.4 \approx 67.38^\circ \]
23. \[ \arcsin .9918 \approx 82.66^\circ \]
24. \[ \arccos .9299 \approx 21.58^\circ \]
25. \[ \arccos .5825 \approx 54^\circ 22' 24'' \]
26. \[ \arctan 2.4 \approx 67^\circ 22' 48'' \]
27. \[ \arcsin .9918 \approx 82^\circ 39' 27'' \]
28. \[ \arccos .9299 \approx 21^\circ 34' 51'' \]
Lesson 5B
1. \( \sin 82^\circ \approx 0.9903 \)
2. \( \tan 14^\circ \approx 0.2493 \)
3. \( \sin 38^\circ \approx 0.6157 \)
4. \( \cos 49^\circ \approx 0.6561 \)
5. \( \cos 58^\circ \approx 0.5299 \)
6. \( \tan 77^\circ \approx 4.3315 \)
7. \( \arcsin 0.175 \approx 1^\circ \)
8. \( \arccos 0.4848 \approx 61^\circ \)
9. \( \arcsin 0.8746 \approx 61^\circ \)
10. \( \arctan 0.4245 \approx 23^\circ \)
11. \( \arctan 1.1918 \approx 50^\circ \)
12. \( \arccos 0.9945 \approx 6^\circ \)
13. \( 47^\circ 36'25" \)
   \( \frac{25'}{1} \times \frac{1'}{60^\circ} = \left( \frac{25}{60} \right)^\circ \approx 42' \)
   \( \frac{36.42'}{1} \times \frac{10'}{60^\circ} = \left( \frac{36.42}{60} \right)^\circ \approx 0.607^\circ \)
   \( 47^\circ 36'25" \approx 47.607^\circ \)
14. \( 53^\circ 28'10" \)
   \( \frac{10'}{1} \times \frac{1'}{60^\circ} = \left( \frac{10}{60} \right)^\circ \approx 0.17' \)
   \( \frac{28.17'}{1} \times \frac{10'}{60^\circ} = \left( \frac{28.17}{60} \right)^\circ \approx 0.470^\circ \)
   \( 53^\circ 28'10" \approx 53.470^\circ \)
15. \( 8^\circ 55' \)
   \( \frac{55'}{1} \times \frac{1'}{60^\circ} = \left( \frac{55}{60} \right)^\circ \approx 0.917^\circ \)
   \( 8^\circ 55' \approx 8.917^\circ \)
16. \( 88^\circ 0'40" \)
   \( \frac{40'}{1} \times \frac{1'}{60^\circ} = \left( \frac{40}{60} \right)^\circ \approx 0.67' \)
   \( \frac{0.67'}{1} \times \frac{1'}{60^\circ} = \left( \frac{0.67}{60} \right)^\circ \approx 0.011^\circ \)
   \( 88^\circ 0'40" \approx 88.011^\circ \)
17. \( 47^\circ 36'25" \approx 47.607^\circ \)
18. \( 53^\circ 28'10" \approx 53.469^\circ \)
19. \( 8^\circ 55' \approx 8.917^\circ \)
20. \( 88^\circ 0'40" \approx 88.011^\circ \)
21. \( \arcsin 0.4700 \approx 28.03^\circ \)
22. \( \arccos 0.9722 \approx 13.54^\circ \)
23. \( \arctan 3.3333 \approx 73.30^\circ \)
24. \( \arcsin 0.9250 \approx 67.67^\circ \)
25. \( \arccos 0.4700 \approx 28^\circ 2'3" \)
26. \( \arccos 0.9722 \approx 13^\circ 32'30" \)
27. \( \arctan 3.3333 \approx 73^\circ 18'00" \)
28. \( \arccos 0.9250 \approx 67^\circ 40'6" \)

Lesson 5C
There are several ways to solve each of these problems. One method will be shown for each.
1. \( \sin 22.5^\circ = \frac{18}{B} \)
   \( B \sin 22.5^\circ = 18 \)
   \( B = \frac{18}{\sin 22.5^\circ} \approx 47 \)
   \( \tan 67.5^\circ = \frac{A}{18} \)
   \( A = \left( \frac{18}{\tan 67.5^\circ} \right) \approx 43.5 \)
   \( \alpha = 90^\circ - 22.5^\circ = 67.5^\circ \)
2. \( \sin 47.3^\circ = \frac{D}{11} \)
   \( D = \left( \frac{11}{\sin 47.3^\circ} \right) \approx 8.1 \)
   \( \sin 42.7^\circ = \frac{C}{11} \)
   \( C = \left( \frac{11}{\sin 42.7^\circ} \right) \approx 7.5 \)
   \( \alpha = 90^\circ - 47.3^\circ = 42.7^\circ \)
3. \( \tan 35.6^\circ = \frac{F}{29} \)
   \( F = \left( \frac{29}{\tan 35.6^\circ} \right) \approx 20.8 \)
   \( \cos 35.6^\circ = \frac{29}{E} \)
   \( E = \left( \frac{29}{\cos 35.6^\circ} \right) \approx 35.7 \)
   \( \alpha = 90^\circ - 35.6^\circ = 54.4^\circ \)
4. \(50^\circ 42'30'' = 50.71^\circ\)
   \[
   \sin 50.71^\circ = \frac{G}{250} \quad G = (250)(\sin 50.71^\circ) \approx 193.5
   \]
   \[
   \cos 50.71^\circ = \frac{H}{250} \quad H = (250)(\cos 50.71^\circ) \approx 158.3
   \]

5. \(\tan \theta = \frac{12}{13} \approx .9231\)
   \[
   \arctan .9231 \approx 42.71^\circ \approx 42^\circ 42'36''
   \]
   \[
   \tan \alpha = \frac{13}{12} \approx 1.0833
   \]
   \[
   \alpha = 90^\circ - \theta = 47.29^\circ \approx 47^\circ 17'24''
   \]

6. \(\tan \theta = \frac{34}{27} \approx 1.2593\)
   \[
   \arctan 1.2593 \approx 51.55^\circ\) or \(51^\circ 32'50''\)
   \[
   \alpha = 90^\circ - 51.55^\circ = 38.45^\circ\) or \(\approx 38^\circ 27''\)

7. \(\sin \theta = \frac{16}{29} \approx .5517\)
   \[
   \arcsin .5517 \approx 33.49^\circ \approx 33^\circ 29'24''
   \]
   \[
   \alpha = 90^\circ - 33.49^\circ = 56.51^\circ \approx 56^\circ 30'36''
   \]

8. \(\cos \theta = \frac{76}{100} = .7600\)
   \[
   \arccos .7600 \approx 40.54^\circ \approx 40^\circ 32'9''
   \]
   \[
   \alpha = 90^\circ - 40.54^\circ = 49.46^\circ \approx 49^\circ 27'36''
   \]

\[\text{Lesson 5D}\]

1. \(\sin 39.1^\circ = \frac{B}{6}\)
   \[
   B = (6)(\sin 39.1^\circ) \approx 3.8
   \]
   \[
   \cos 39.1^\circ = \frac{A}{6}
   \]
   \[
   A = (6)(\cos 39.1^\circ) \approx 4.7
   \]
   \[
   \alpha = 90^\circ - 39.1^\circ = 50.9^\circ
   \]

2. \(\cos 42.25^\circ = \frac{20}{D}\)
   \[
   D = \frac{20}{\cos 42.25^\circ} \approx 27.0
   \]
   \[
   \sin 42.25^\circ = \frac{C}{27.0}
   \]
   \[
   C = (27)(\sin 42.25^\circ) \approx 18.2
   \]
   \[
   \theta = 42.25^\circ \approx 42.25^\circ
   \]
   \[
   \alpha = 90^\circ - 42.25^\circ = 47.75^\circ
   \]

3. \(\tan 28^\circ = \frac{E}{57}\)
   \[
   E = (57)(\tan 28^\circ) \approx 30.3
   \]
   \[
   \sin 62^\circ = \frac{57}{F}
   \]
   \[
   F = \frac{57}{\sin 62^\circ} \approx 64.6
   \]
   \[
   \alpha = 90^\circ - 62^\circ = 28^\circ
   \]

4. \(90^\circ - 25.73^\circ = 64.27^\circ\)
   \[
   \cos 64.27^\circ = \frac{18.3}{H}
   \]
   \[
   H = \frac{18.3}{\cos 64.27^\circ} \approx 42.2
   \]
   \[
   \sin 64.27^\circ = \frac{G}{42.2}
   \]
   \[
   G = (42.2)(\sin 64.27^\circ) \approx 38.0^\circ
   \]
   \[
   \theta = 25^\circ 44'\approx 25.73^\circ
   \]
   \[
   \alpha = 90^\circ - 25.73^\circ = 64.27^\circ
   \]

5. \(\tan \theta = \frac{6.2}{5.4} \approx 1.1481\)
   \[
   \arctan 1.1481 \approx 48.95^\circ
   \]
   \[
   \theta \approx 48.95^\circ \approx 48^\circ 56'38"
   \]
   \[
   \alpha = 90^\circ - 48.95^\circ = 41.05^\circ \approx 41^\circ 3'
   \]

6. \(\tan \theta = \frac{40.5}{28.7} \approx 1.4111\)
   \[
   \arctan 1.4111 \approx 54.68^\circ
   \]
   \[
   \theta \approx 54.68^\circ = 54^\circ 40'34"
   \]
   \[
   \alpha = 90^\circ - 54.68^\circ = 35.32^\circ \approx 35^\circ 19'12"
   \]

7. \(\cos \theta = \frac{39}{63} \approx .6190\)
   \[
   \arccos .6190 \approx 51.76^\circ \approx 51^\circ 45'25''
   \]
   \[
   \alpha = 90^\circ - 51.76^\circ = 38.24^\circ \approx 38^\circ 14'24"
   \]

8. \(\sin \theta = \frac{3.8}{7.9} \approx .4810\)
   \[
   \arcsin .4810 \approx 28.75^\circ
   \]
   \[
   \theta \approx 28.75^\circ \approx 28^\circ 45'03"
   \]
   \[
   \alpha = 90^\circ - 28.75^\circ = 61.25^\circ \approx 61^\circ 15'