We’ve worked on a fraction of one and a fraction of a number; now we’ll tackle a fraction of a fraction. This kind of problem can be the hardest to understand or think through, but it is the easiest to do using a formula. Find $\frac{2}{5}$ of $\frac{1}{3}$. Notice that we don’t say, “two-fifths times one-third.” Even though it is the same as multiplying two fractions, I want to relate it to a fraction of a number and a fraction of one. To stress this relationship, we read it “two-fifths of one-third.” This is also the language used in most word problems. Be sure to read the note about word problems on lesson practice 9A in the student text.

**Example 1**

Find $\frac{2}{5}$ of $\frac{1}{3}$ (two-fifths of one-third).

**Step 1** Start with $\frac{1}{3}$.

**Step 2** Divide into 5 equal parts.
Step 3  Count 2 of those parts.

Pull up the pink insert to do this.

So \( \frac{2}{5} \) of \( \frac{1}{3} \) is \( \frac{2}{15} \).

Example 2

Find \( \frac{2}{3} \) of \( \frac{5}{6} \) (two-thirds of five-sixths).

Step 1  Start with \( \frac{5}{6} \).

Step 2  Divide into 3 equal parts.

Step 3  Count 2 of those parts.

Pull up the violet insert to do this.

So \( \frac{2}{3} \) of \( \frac{5}{6} \) is \( \frac{10}{18} \).
Another way to understand the formula for a fraction of a fraction (multiplying fractions) is shown in figure 1, which reveals the finished answer to example 2. To make the “factors” clearer I placed a white piece over the answer so you can see just the two factors of the two rectangles. I hope this helps you “see” numerator times numerator, divided by denominator times denominator.

Figure 1

The whole square is an illustration of $\frac{2}{3} \times \frac{5}{6}$, which is a multiplication problem. In this problem we can see the two smaller problems. The numerator problem has the factors $2 \times 5$. The denominator problem has the factors $3 \times 6$. This exercise reveals the formula for multiplying two fractions:

$$\frac{2}{3} \times \frac{5}{6} = \frac{2 \times 5}{3 \times 6} = \frac{10}{18}$$

\[ \frac{\text{Numerator times numerator (shaded rectangle)}}{\text{Denominator times denominator (total # of parts)}} \]
MENTAL MATH
Here are some more mental math problems for you to read aloud to your student. Try a few at a time, going slowly at first.

1. Four plus five, times two, divided by three, equals ? (6)
2. Thirty-six divided by four, minus two, times five, equals ? (35)
3. Sixty-six divided by six, plus four, divided by three, equals ? (5)
4. Forty-eight divided by eight, times six, plus two, equals ? (38)
5. Six plus seven, minus four, times nine, equals ? (81)
6. Nineteen minus three, divided by four, times seven, equals ? (28)
7. Nine times five, plus three, divided by six, equals ? (8)
8. Twenty-one divided by seven, plus one, times five, equals ? (20)
9. Three times four, divided by two, times nine, equals ? (54)
10. Seven times six, plus two, divided by 11, equals ? (4)
LESSON PRACTICE

Find a fraction of a fraction. These can all be built with the overlays. The first one is done for you.

1. \( \frac{2}{4} \) of \( \frac{3}{6} = \frac{2 \times 3}{4 \times 6} = \frac{6}{24} \)

2. \( \frac{3}{5} \) of \( \frac{3}{4} = \) __________

3. \( \frac{1}{3} \) of \( \frac{1}{2} = \) __________

4. \( \frac{4}{6} \) of \( \frac{2}{5} = \) __________

5. \( \frac{2}{3} \) of \( \frac{1}{4} = \) __________

6. \( \frac{1}{5} \) of \( \frac{5}{6} = \) __________

7. \( \frac{4}{5} \) of \( \frac{2}{6} = \) __________

8. \( \frac{1}{6} \) of \( \frac{1}{3} = \) __________

9. \( \frac{1}{2} \) of \( \frac{2}{6} = \) __________
Multiply the fractions. (Find a fraction of a fraction.) These can all be built with the overlays.

10. \( \frac{3}{5} \times \frac{2}{5} = \) 
11. \( \frac{3}{6} \times \frac{2}{3} = \)

12. \( \frac{2}{4} \times \frac{1}{5} = \)
13. \( \frac{3}{4} \times \frac{4}{6} = \)

14. \( \frac{2}{5} \times \frac{1}{2} = \)
15. \( \frac{4}{6} \times \frac{1}{3} = \)

Always read fraction word problems carefully and think about what is happening. Multiplication problems involving fractions usually use the word “of,” but you should not assume that the word “of” somewhere in the problem automatically indicates multiplication. Read for meaning, don’t just look for key words. All of the examples on lesson practice A, B, and C are multiplication of fractions.

16. A recipe calls for \( \frac{1}{3} \) of a cup of butter. If Matthew is making \( \frac{1}{4} \) of the recipe, how much butter should he use?

17. One-half of the people at the picnic went home sick, but only one-fifth of them were seriously ill. What part of the whole group was seriously ill?

18. Mom left \( \frac{1}{3} \) of a pie for Chucky to eat, but since he was not very hungry, he ate only \( \frac{1}{5} \) of what was there. What part of a whole pie did Chucky eat?
Multiply (fraction of a fraction).

1. \( \frac{2}{5} \) of \( \frac{1}{5} \) = ___

2. \( \frac{5}{6} \times \frac{1}{4} \) = ___

3. \( \frac{5}{9} \times \frac{1}{2} \) = ___

Add or subtract.

4. \( \frac{1}{7} + \frac{1}{3} \) = ___

5. \( \frac{2}{3} - \frac{1}{7} \) = ___

6. \( \frac{1}{2} + \frac{2}{5} + \frac{7}{10} \) = ___

Use the rule of four to make denominators the same, then compare the fractions.

7. \( \frac{4}{6} \bigcirc \frac{1}{7} \)

8. \( \frac{2}{3} \bigcirc \frac{5}{8} \)

9. \( \frac{1}{9} \bigcirc \frac{2}{11} \)
Fill in the missing numbers in the numerators or denominators to make equivalent fractions.

10. \( \frac{2}{7} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{28}{\quad} \)

11. \( \frac{\quad}{\quad} = \frac{2}{18} = \frac{\quad}{\quad} = \frac{4}{\quad} \)

QUICK REVIEW

Here is a chance to review multiplying a two-digit number by a three-digit number. The first one is done for you.

Estimate, then multiply to find the exact answer.

12. \( 623 \times 45 \)

\[
\begin{array}{c}
& 623 \\
\times & 45 \\
\hline
11 & 600 \\
30 & 000 \\
1 & 2482 \\
\hline
28035
\end{array}
\]

13. \( 179 \times 57 \)

\[
\begin{array}{c}
179 \\
\times & 57 \\
\hline
11 & 700 \\
30 & 000 \\
1 & 2482 \\
\hline
28035
\end{array}
\]

\( \) (This is 5 tens x 6 hundreds.)

14. \( 902 \times 11 \)

\[
\begin{array}{c}
902 \\
\times 11 \\
\hline
9922
\end{array}
\]

15. Mom is making \( \frac{1}{3} \) of a recipe that calls for \( \frac{2}{3} \) of a cup of flour. How much flour should she use?

16. If there are 365 days in one year, how many days are there in 25 years? (Don’t worry about leap years.)

17. Nancy needs \( \frac{1}{2} \) cup of honey for bread, \( \frac{1}{3} \) cup for cookies, and \( \frac{1}{4} \) cup for fruit salad. How much honey does she need altogether?

18. Ivan spotted 36 birds Saturday morning at Hawk Mountain. Three-fourths of them were hawks. How many hawks did he see?
Multiply (fraction of a fraction).

1. \( \frac{1}{4} \) of \( \frac{1}{3} \) = 

2. \( \frac{3}{4} \times \frac{2}{5} \) = 

3. \( \frac{2}{7} \times \frac{1}{4} \) = 

4. \( \frac{2}{5} \) of \( \frac{3}{7} \) = 

5. \( \frac{3}{5} \times \frac{1}{6} \) = 

6. \( \frac{2}{4} \times \frac{1}{3} \) = 

Add or subtract.

7. \( \frac{3}{11} + \frac{1}{4} \) = 

8. \( \frac{4}{5} - \frac{1}{6} \) = 

9. \( \frac{1}{7} + \frac{2}{3} + \frac{1}{2} \) = 
Use the rule of four to make denominators the same, then compare the fractions.

10. \( \frac{2}{3} \) \( \bigcirc \) \( \frac{5}{8} \)
11. \( \frac{3}{4} \) \( \bigcirc \) \( \frac{9}{12} \)
12. \( \frac{5}{6} \) \( \bigcirc \) \( \frac{7}{9} \)

Estimate, then multiply to find the exact answer.

13. \( 612 \times 54 \)
14. \( 124 \times 36 \)
15. \( 957 \times 13 \)

16. One-third of the customers at the ice cream store bought vanilla and two-fifths of them bought chocolate. What part of the customers bought vanilla or chocolate?

17. What is the perimeter of a triangle whose sides measure 10 feet, 12 feet, and 12 feet?

18. Three-eighths of the guests at the picnic ate hamburgers. One-half of those people had mustard on their hamburgers. What part of the people at the picnic had hamburgers with mustard?

19. If there were 48 people at the picnic (#18), how many had hamburgers? How many had hamburgers with mustard?

20. Each of the 48 people at the picnic contributed $15 for food and other expenses. How much money was collected?
20. \[25 + 4 = \frac{61}{4}\] chocolates
Remember, equivalent answers to addition problems are correct.

Lesson Practice 9A
1. done
2. \[\frac{3 \times 3}{5 \times 4} = \frac{9}{20}\]
3. \[\frac{1 \times 1}{3 \times 2} = \frac{1}{6}\]
4. \[\frac{4 \times 2}{6 \times 5} = \frac{8}{30}\]
5. \[\frac{2 \times 1}{3 \times 4} = \frac{2}{12}\]
6. \[\frac{1 \times 5}{5 \times 6} = \frac{5}{30}\]
7. \[\frac{4 \times 2}{5 \times 6} = \frac{8}{30}\]
8. \[\frac{1 \times 1}{6 \times 3} = \frac{1}{18}\]
9. \[\frac{1 \times 2}{2 \times 6} = \frac{2}{12}\]
10. \[\frac{3 \times 2}{5 \times 5} = \frac{6}{25}\]
11. \[\frac{3 \times 2}{6 \times 3} = \frac{6}{18}\]
12. \[\frac{2 \times 1}{4 \times 5} = \frac{2}{20}\]
13. \[\frac{3 \times 4}{4 \times 6} = \frac{12}{24}\]
14. \[\frac{2 \times 1}{5 \times 2} = \frac{2}{10}\]
15. \[\frac{4 \times 1}{6 \times 3} = \frac{4}{18}\]
16. \[\frac{1 \times 1}{3 \times 4} = \frac{1}{12}\]
17. \[\frac{1 \times 1}{2 \times 5} = \frac{1}{10}\]
18. \[\frac{1 \times 1}{3 \times 5} = \frac{1}{15}\]

Lesson Practice 9B
1. \[\frac{2 \times 3}{6 \times 5} = \frac{6}{30}\]
2. \[\frac{2 \times 2}{3 \times 2} = \frac{2}{6}\]
3. \[\frac{3 \times 2}{8 \times 4} = \frac{6}{32}\]
4. \[\frac{1 \times 1}{4 \times 5} = \frac{1}{20}\]
5. \[\frac{1 \times 2}{6 \times 3} = \frac{2}{18}\]
6. \[\frac{1 \times 4}{3 \times 5} = \frac{4}{15}\]
7. \[\frac{4 \times 3}{5 \times 6} = \frac{12}{30}\]
8. \[\frac{1 \times 5}{2 \times 6} = \frac{5}{12}\]
9. \[\frac{3 \times 3}{4 \times 4} = \frac{9}{16}\]
10. \[\frac{2 \times 4}{5 \times 6} = \frac{8}{30}\]
11. \[\frac{1 \times 1}{7 \times 2} = \frac{1}{14}\]
12. \[\frac{3 \times 5}{2 \times 6} = \frac{15}{30}\]
13. \[\frac{3 \times 1}{10 \times 4} = \frac{3}{40}\]
14. \[\frac{1 \times 1}{4 \times 2} = \frac{1}{8}\]
15. \[\frac{7 \times 1}{8 \times 9} = \frac{7}{72}\]
16. \[\frac{2 \times 1}{4 \times 4} = \frac{3}{16}\]
17. \[\frac{2 \times 3}{3 \times 5} = \frac{6}{15}\]
18. \[\frac{1 \times 1}{5 \times 5} = \frac{1}{25}\]

Lesson Practice 9C
1. \[\frac{3 \times 2}{6 \times 5} = \frac{6}{30}\]
2. \[\frac{2 \times 2}{9 \times 3} = \frac{4}{27}\]
LESSON PRACTICE 9C - SYSTEMATIC REVIEW 9E

Systematic Review 9D

1. \( \frac{2 \times 1}{5 \times 5} = \frac{2}{25} \)
2. \( \frac{5 \times 1}{6 \times 4} = \frac{5}{24} \)
3. \( \frac{5 \times 1}{9 \times 2} = \frac{5}{18} \)
4. \( \frac{3 + 7}{21 + 21} = \frac{10}{21} \)
5. \( \frac{14 - 3}{21 - 21} = \frac{11}{21} \)
6. \( \frac{5}{10} + \frac{4}{10} + \frac{7}{10} = \frac{16}{10} \) or \( \frac{1}{6} \)
7. \( \frac{28}{42} > \frac{6}{42} \)
8. \( \frac{16}{24} > \frac{15}{24} \)
9. \( \frac{11}{99} \leq \frac{18}{99} \)
10. \( \frac{2}{7} = \frac{4}{14} = \frac{6}{21} = \frac{8}{28} \)
11. \( \frac{1}{9} = \frac{2}{18} = \frac{3}{27} = \frac{4}{36} \)
12. done
13. \((200) \times (60) = (12,000)\)
179 \times 57 = 10,203
14. \((900) \times (10) = (9,000)\)
902 \times 11 = 9,922
15. \(\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}\) of a cup
16. \(365 \times 25 = 9,125\) days
17. \(\frac{3}{6} + \frac{2}{6} = \frac{5}{6}\)
\(\frac{5}{6} + \frac{1}{4} = \frac{20}{24} + \frac{6}{24} = \frac{26}{24}\)
of \(\frac{1}{24}\) cups
18. \(36 \times 4 = 9\)
\(9 \times 3 = 27\) hawks

Systematic Review 9E

1. \(\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}\)
2. \(\frac{7}{8} \times \frac{2}{5} = \frac{14}{40}\)
3. \(\frac{3}{3} \times \frac{3}{5} = \frac{9}{15}\)
4. \(\frac{27}{36} + \frac{4}{36} = \frac{31}{36}\)
5. \(\frac{10}{15} - \frac{6}{15} = \frac{4}{15}\)
6. \(\frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1\)
\(1 + \frac{2}{8} = 1\frac{2}{8}\) or \(1\frac{1}{4}\)
7. \(\frac{9}{27} = \frac{9}{27}\)
8. \(\frac{16}{40} > \frac{15}{40}\)
9. \(\frac{50}{70} > \frac{49}{70}\)
11. \( (300) \times (40) = (12,000) \)
\[ 254 \times 35 = 8,890 \]
12. \( (600) \times (30) = (18,000) \)
\[ 563 \times 26 = 14,638 \]
13. \( 107 \div 6 = 17 \frac{5}{6} \)
14. \( 395 \div 8 = 49 \frac{3}{8} \)
15. \( 459 \div 2 = 229 \frac{1}{2} \)
16. \( \frac{1}{2} \times \frac{4}{5} = \frac{4}{10} \) of the chores
17. \[ \frac{5}{30} \div \frac{6}{30} = \frac{11}{30} \]
\[ \frac{11}{30} + \frac{1}{4} = \frac{44}{120} + \frac{30}{120} = \frac{74}{120} \] of the job
18. \( 250 \times 51 = 12,750 \) in
19. \( 12 + 17 + 28 = 57 \) in
20. \( 235 \div 5 = 47; \ 47 \times 3 = 141 \)
   jelly beans

Systematic Review 9F
1. \( \frac{2}{5} \times \frac{2}{3} = \frac{4}{15} \)
2. \( \frac{1}{3} \times \frac{2}{5} = \frac{2}{15} \)
3. \( \frac{1}{4} \times \frac{4}{15} = \frac{4}{60} \)
4. \( \frac{10}{15} \div \frac{3}{15} = \frac{13}{15} \)
5. \( \frac{16}{40} \div \frac{15}{40} = \frac{1}{1} \)
6. \( \frac{5}{20} \div \frac{12}{20} = \frac{17}{20} \)
\[ \frac{17}{20} \div \frac{2}{3} = \frac{51}{60} + \frac{40}{60} = \frac{91}{60} \] or \( 1 \frac{31}{60} \)
7. \( \frac{40}{48} \div \frac{24}{48} \)
8. \( \frac{27}{45} \div \frac{20}{45} \)
9. \( \frac{24}{32} \div \frac{24}{32} \)
10. \( (600) \times (60) = (36,000) \)
\[ 558 \times 62 = 34,596 \]
11. \( (400) \times (80) = (32,000) \)
\[ 407 \times 83 = 33,781 \]
12. \( (300) \times (10) = (3,000) \)
\[ 349 \times 12 = 4,188 \]
13. \( 128 \div 7 = 18 \frac{2}{7} \)
14. \( 471 \div 3 = 157 \)
15. \( 298 \div 5 = 59 \frac{3}{5} \)
16. \( \frac{3}{2} \times \frac{1}{3} = \frac{3}{4} \text{ in} \)
17. \( \frac{4}{5} \div \frac{1}{3} = \frac{12}{5} \div \frac{15}{15} = \frac{7}{15} \text{ mile} \)
18. \( \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \) of 12 = 2 people
19. \( 13 + 19 + 26 = 58 \) in
20. \( 30 \div 2 = 15 \)
\[ 15 \times 1 = 15 \text{ days} \]

Lesson Practice 10A
1. done
2. done
3. \( \frac{15}{24} \div \frac{16}{24} = \frac{15}{16} \)
4. \( \frac{12}{16} \div \frac{8}{16} = \frac{12}{1} \div \frac{8}{8} = 1 \frac{4}{1} \text{ or } 1 \frac{4}{8} \text{ or } 1 \frac{4}{16} \)
5. \( \frac{4}{8} \div \frac{2}{8} = \frac{4}{2} = 2 \)
6. \( \frac{1}{2} \div \frac{8}{16} = \frac{8}{1} \div \frac{2}{1} = 8 \div 2 = 4 \text{ times} \)
7. \( \frac{2}{3} \div \frac{1}{18} = \frac{12}{1} \div \frac{3}{1} = 12 \div 3 = 4 \text{ pieces} \)
8. \( \frac{6}{8} \div \frac{1}{32} = \frac{24}{4} \div \frac{8}{32} = \frac{24}{8} \div \frac{32}{32} = 3 \text{ people} \)

Lesson Practice 10B
1. \( \frac{12}{20} \div \frac{5}{20} = \frac{12}{1} \div \frac{5}{5} = 12 \div 5 = \frac{1}{2} \text{ or } 2 \frac{2}{5} \)
2. \( \frac{18}{24} \div \frac{4}{24} = \frac{18}{1} \div \frac{4}{4} = 18 \div 4 = \frac{1}{1} \text{ or } 4 \frac{2}{4} \)
3. \( \frac{3}{12} \div \frac{4}{12} = \frac{3}{1} \div \frac{4}{4} = 3 \div 4 \)
4. \( \frac{16}{24} \div \frac{15}{24} = \frac{16}{1} \div \frac{15}{15} = \frac{16}{15} \text{ or } 1 \frac{1}{15} \)
5. \( \frac{5}{20} \div \frac{16}{20} = \frac{5}{1} \div \frac{16}{16} = \frac{5}{16} \)
### Test 9

1. \[
\frac{1}{4} \text{ of } \frac{1}{3} = \frac{1}{12}
\]
2. \[
\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}
\]
3. \[
\frac{2}{7} \times \frac{1}{4} = \frac{2}{28}
\]
4. \[
\frac{2}{5} \text{ of } \frac{3}{7} = \frac{6}{35}
\]
5. \[
\frac{3}{5} \times \frac{1}{6} = \frac{3}{30}
\]
6. \[
\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}
\]
7. \[
\frac{12}{44} + \frac{11}{44} = \frac{23}{44}
\]
8. \[
\frac{24}{30} - \frac{5}{30} = \frac{19}{30}
\]
9. \[
\frac{3}{21} + \frac{14}{21} = \frac{17}{21}
\]
10. \[
\frac{16}{24} > \frac{15}{24}
\]
11. \[
\frac{36}{48} = \frac{3}{4}
\]
12. \[
\frac{45}{54} > \frac{42}{54}
\]
13. \[
(600) \times (50) = (30,000)
\]
   \[
612 \times 54 = 33,048
\]
14. \[
(100) \times (40) = (4,000)
\]
   \[
124 \times 36 = 4,464
\]
15. \[
(1000) \times (10) = (10,000)
\]
   \[
957 \times 13 = 12,441
\]
16. \[
\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}
\]
   \[
\text{of the customers}
\]
17. \[
10 + 12 + 12 = 34 \text{ ft}
\]
18. \[
\frac{3}{8} \times \frac{1}{2} = \frac{3}{16}
\]
   \[
\text{of the people}
\]
19. \[
\frac{2}{3} \text{ of 48:}
\]
   \[
48 + 8 = 6; 6 \times 3 = 18 \text{ had burgers}
\]
   \[
\frac{3}{16} \text{ of 48:}
\]
   \[
48 + 16 = 3; 3 \times 3 = 9 \text{ had mustard}
\]
20. \[
48 \times 15 = 720
\]

### Test 10

1. \[
\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}
\]
2. \[
\frac{15}{24} \div \frac{16}{24} = \frac{15}{16} \div \frac{1}{16} = 15
\]
3. \[
\frac{4}{8} + \frac{2}{8} = \frac{4+2}{8} = \frac{2}{1}
\]
4. \[
\frac{3}{12} + \frac{4}{12} = \frac{3+4}{12} = \frac{7}{12}
\]
5. \[
\frac{27}{45} \div \frac{10}{45} = \frac{27}{10} \div \frac{1}{10} = 27 \text{ or } 2.7
\]
6. \[
\frac{12}{15} \times \frac{10}{15} = \frac{12}{10} = \frac{12}{10} \text{ or } 1\frac{2}{10}
\]
7. \[
\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}
\]
8. \[
\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}
\]
9. \[
\frac{2}{9} \times \frac{1}{1} = \frac{2}{18}
\]
10. \[
\frac{32}{56} + \frac{21}{56} = \frac{53}{56}
\]
11. \[
\frac{20}{36} - \frac{9}{36} = \frac{11}{36}
\]
12. \[
\frac{4}{10} + \frac{3}{10} = \frac{12}{10} \text{ or } 1\frac{2}{10}
\]
13. \[
(500) \times (40) = (10)
\]
14. \[
(900) \times (30) = (30)
\]
15. \[
(600) \times (60) = (10)
\]