

Lesson 7 Slope-Intercept Formula

Terms Two new words that describe what we've been doing in graphing lines are slope and intercept. The slope is referred to as "m" (a mountain has slope and starts with "m"), and intercept as "b" (a bee that intercepts your leg with his stinger!) The slope-intercept formula is $Y = mX + b$. In $Y = 2X + 3$, 2 is the slope and 3 is the intercept.

Slope Slope is the $\frac{\text{up dimension}}{\text{over dimension}}$, or the rise over the run. ($-\frac{\text{rise}}{\text{run}}$) In our example of the bread baking, for every one hour (over) we were able to bake three loaves (up). So for every hour we move over to the right one space and up three spaces. We continue to do this, and when we connect two or more points, we have a line that "slopes" up. We describe the slope as $\frac{3 \text{ up}}{1 \text{ over}}$ or 3. If we made five loaves each hour, it would be a steeper slope, $\frac{5 \text{ up}}{1 \text{ over}}$ or 5. The higher the slope, the steeper the line. If it takes four hours to make one loaf of bread, the slope would be $\frac{1 \text{ up}}{4 \text{ over}}$ or $1/4$. It would be over four spaces and up one space. Look at Figure 1 for slopes of $m = 3$, $m = 5$, and $m = 1/4$.

Figure 1

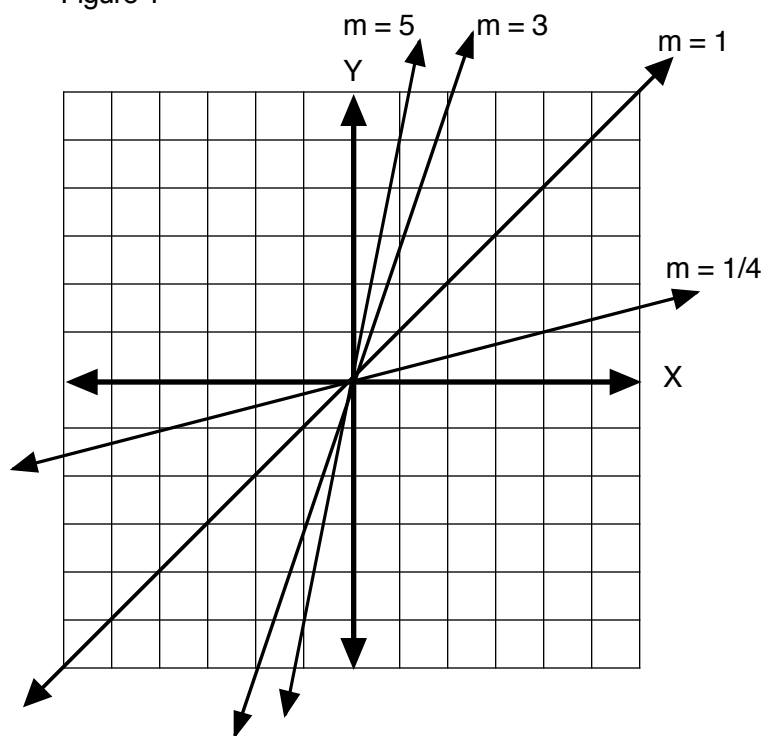
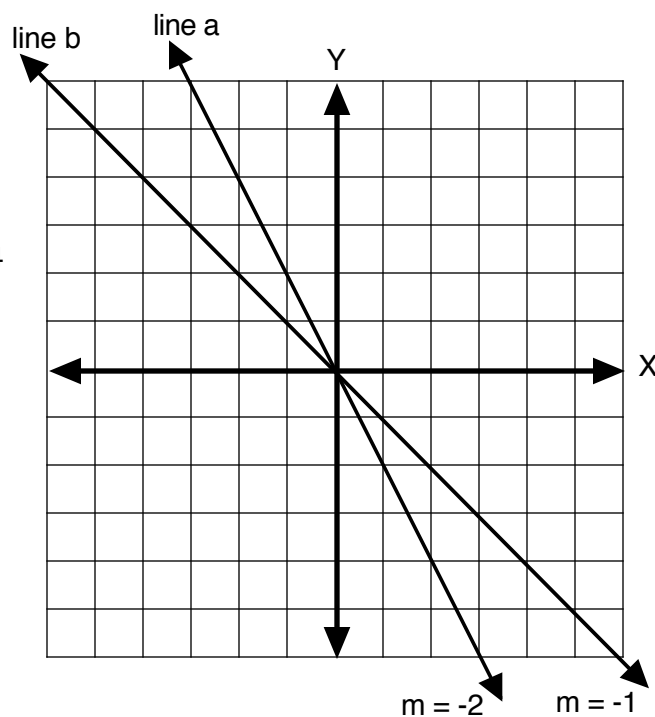


Figure 2



Negative Slope You can also have negative slopes. An example would be the business man who loses two dollars each day. For every one day, minus two dollars. The slope is over one and down two (the opposite of up, because it is minus). It will look like line a in Figure 2 ($Y = -2X + 0$, or $Y = -2X$).

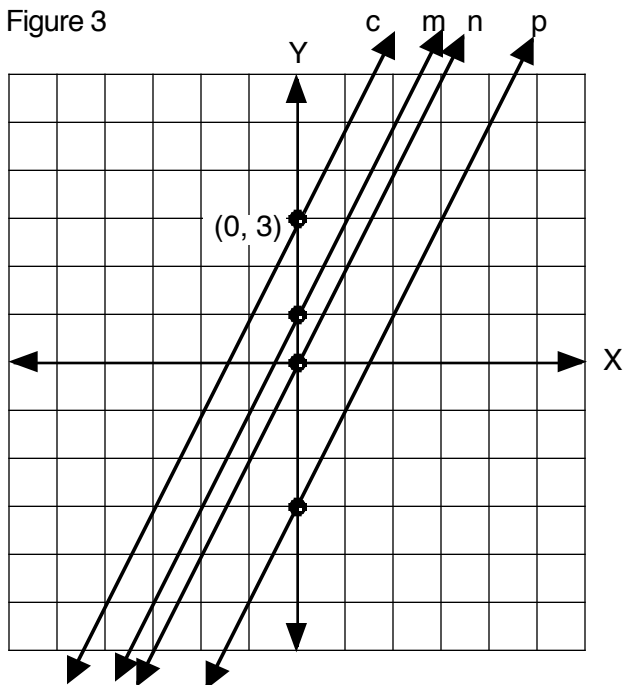
Days	Money
1	-2
2	-4
3	-6

Line b in Figure 2 is an example of losing one dollar per day ($Y = -1X$, or $Y = -X$).

Intercepts There are an infinite number of lines that have the same slope. When you add the y-intercept, you narrow it down to one specific line. Notice in Figure 3 there are 4 lines. Each of these lines has a slope of $2/1$ or 2 . What makes each line different is where it intercepts y-axis.

Notice that the lines have the same slope. What differentiates each line is the position on the graph which is described by the y-intercept.

Figure 3

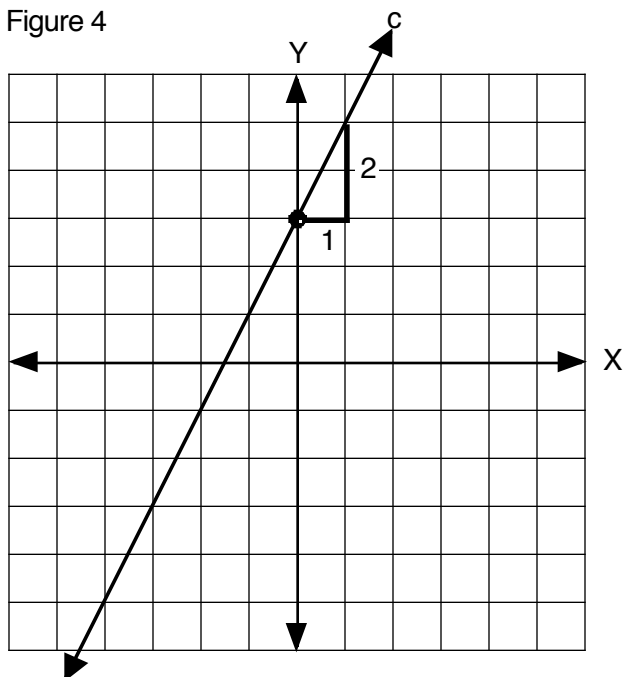


The intercept of line c is the point $(0, 3)$. All along the Y-axis, the coordinate for X is 0. The point $(0, 3)$ tells us that $X = 0$ and $Y = 3$. So our Y intercept is found whenever X is 0, and in this case it is 3. Now that we know the intercept we need to find the slope.

The intercepts of lines m, n, and p are:
line m $(0, 1)$, line n $(0, 0)$, line p $(0, -3)$.

The slope intercept formulas for these 3 lines are: line m: $Y = 2X + 1$, line n: $Y = 2X + 0$ or $Y = 2X$, line p: $Y = 2X - 3$.

Figure 4

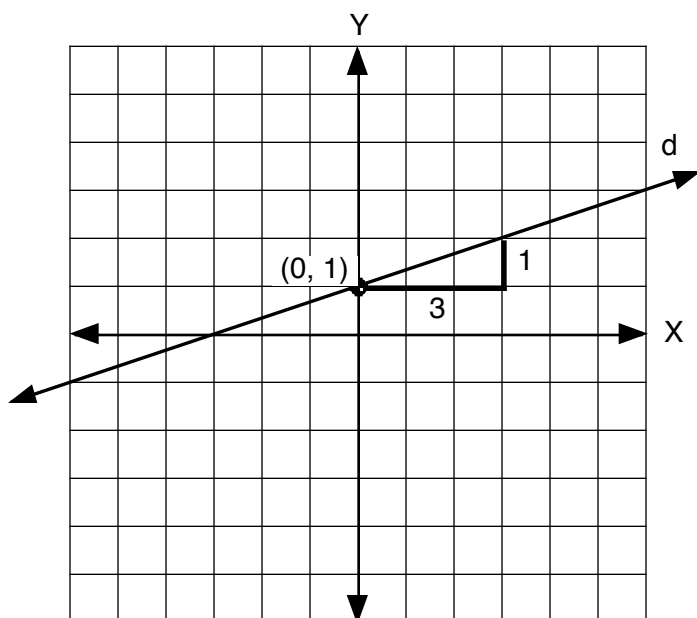


By making a right triangle we may read the rise and the run, or the over and the up. In the triangle on Figure 4, beginning at the intercept, we see the rise (up) is 2 and the run (over) is 1. The slope is $2/1$ or 2 .

Since the slope is $2/1$ or 2 , then the slope-intercept representing line c is $Y = 2X + 3$.

Figure 5

Find the slope and intercept of line d.



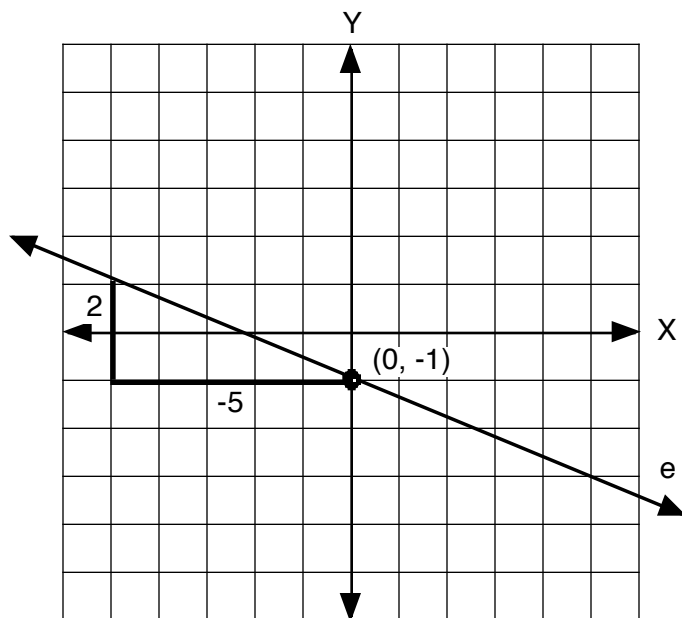
The intercept (also known as the Y-intercept) of line d is the point $(0, 1)$. All along the Y-axis, the coordinate for X is 0. The point $(0, 1)$ tells us that $X = 0$ and $Y = 1$. So our Y intercept is found whenever X is 0, therefore the y-intercept is 1.

By making a right triangle we may read the rise and the run, or the over and the up. In the triangle on Figure 5, beginning at the intercept, we see the rise (up) is 1 and the run (over) is 3. The slope is $1/3$.

Since the slope is $1/3$, and the intercept is 1, then the slope intercept representing line c is $Y = 1/3X + 1$.

Figure 6

Find the slope and intercept of line e.



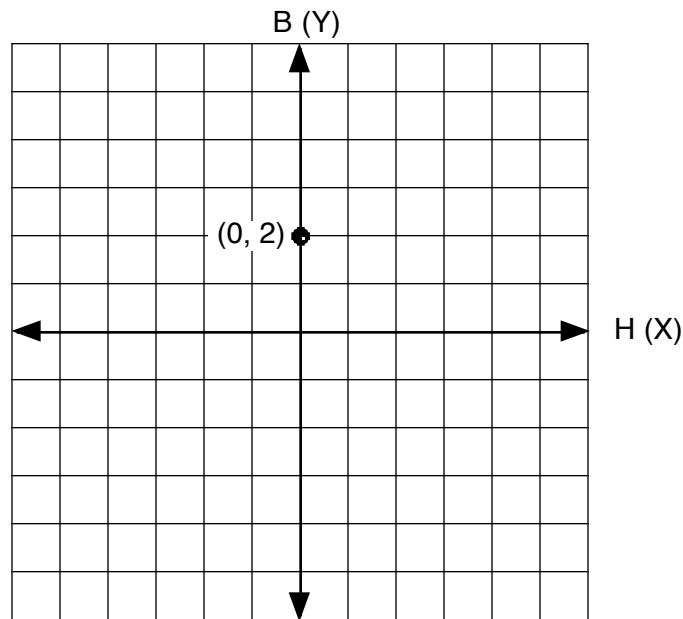
The intercept (also known as the Y-intercept) of line d is the point $(0, -1)$. The point $(0, -1)$ tells us that $X = 0$ and $Y = -1$. So our Y intercept is found whenever X is 0, therefore the y-intercept is -1.

By making a right triangle we may read the rise and the run, or the over and the up. In the triangle on Figure 6, beginning at the intercept, we see the rise (up) is 2 and the run (over) is -5 since it is moving to the left. The slope is $2/-5$ or $-2/5$.

Since the slope is $-2/5$, and the intercept is -1, then the slope intercept representing line c is $Y = -2/5X - 1$.

Graphing a Line The inverse of finding the slope-intercept formula of a line is graphing a line when you are given the slope-intercept formula. In Figure 7, we are given the formula as $B = 3H + 2$. The first step is to find the intercept. Since the coordinate of X (in this case H) is 0 at the intercept, you can make $H = 0$ in $B = 3H + 2$. Substituting, $B = 3(0) + 2$ and $B = 2$. So the intercept is 2. Let's draw that point on the graph.

Figure 7

Graph $B = 3H + 2$ 

We know the slope (or how much bread per hour) is the coefficient of H. It is 3 or $3/1$ and has a rise of 3 and a run of 1. Beginning at the intercept, we draw a triangle with a run (over) 1 and a rise (up) 3. This brings us to the point (1, 5). If we connect the 2 points we have a line representing $B = 3H + 2$ or $Y = 3X + 2$.

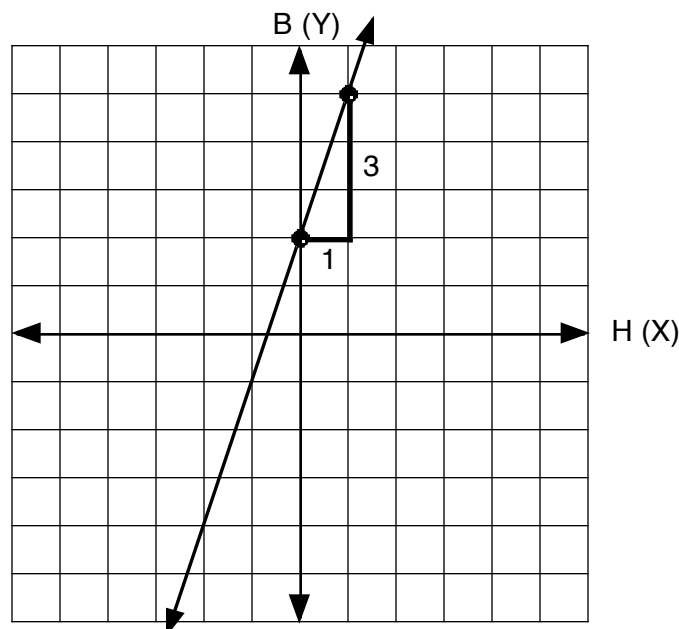
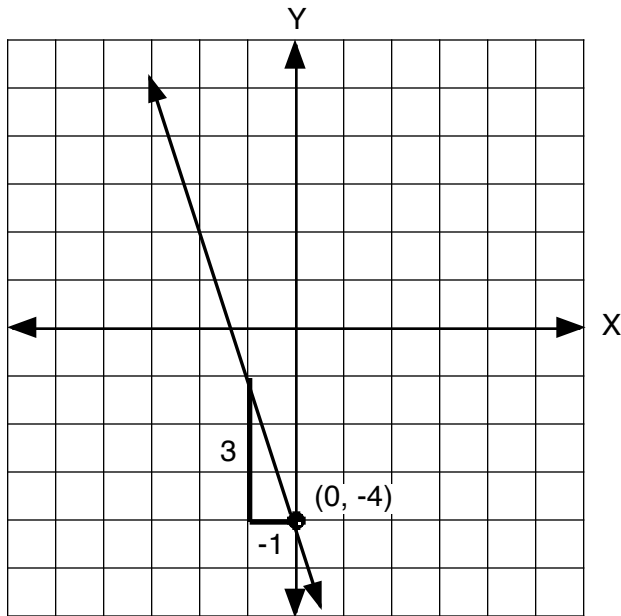


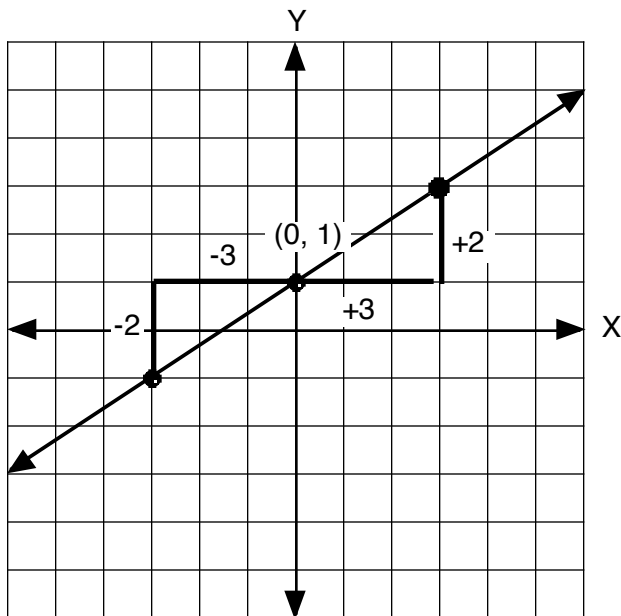
Figure 8

Graph $Y = -3X - 4$ 

To find the intercept, we make X equal to 0. So then the intercept is -4 , and we plot the point $(0, -4)$.

We can see the slope is -3 since it is the coefficient, or what is multiplied, times X . Beginning at $(0, -4)$ we construct a triangle with a rise of 3 and a run of -1 (we could have made this -3 and $+1$ as it would still produce a slope of -3). After connecting the points $(0, -4)$ and $(-1, -1)$ we have a graphic representation of the line described as $Y = -3X - 4$.

Figure 9

Graph $Y = 2/3X + 1$ 

To find the intercept, we make X equal to 0. So then the intercept is 1, and we plot the point $(0, 1)$.

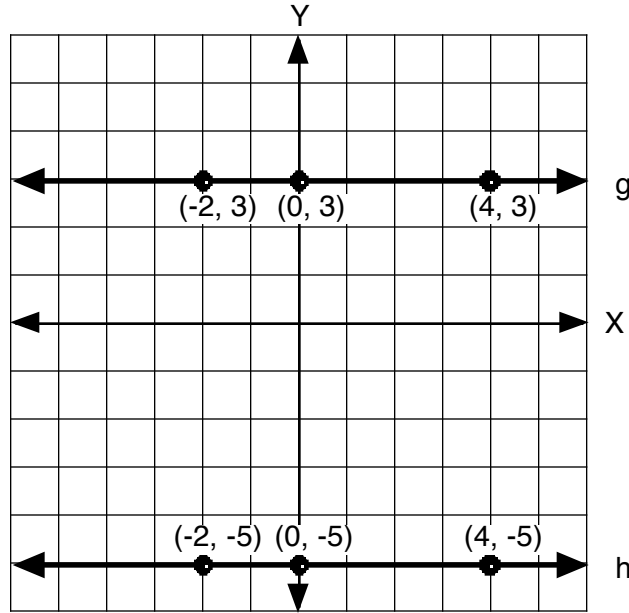
We can see the slope is $2/3$ since it is the coefficient, or what is multiplied, times X . Beginning at $(0, 1)$ we construct a triangle with a rise of 2 and a run of 3. In the graph notice that if we went to the left to make our triangle, the rise is (-2) and the run is (-3) . And $-2/-3$ still produces a slope of $+2/3$.

After connecting the points $(0, 1)$ and $(-3, 1)$ or $(0, 1)$ and $(3, 3)$ we have a graphic representation of the line described as $Y = 2/3X + 1$.

Horizontal and Vertical Lines There are lines that don't seem to have any slope at all. They are either horizontal or vertical. In figure 10 notice line g. Observe that every point along that line has one coordinate the same. All three points have an Y coordinate of 3. In fact every point along this line also has a Y coordinate of 3. So we say the equation of this line is $Y = 3$. What do you think the equation of line h is? If you said $Y = -5$, you are correct.

The slope of line g is $\frac{\text{rise}}{\text{run}}$ which is $\frac{0}{4}$ between points the $(0, 3)$ and $(4, 3)$ and $0 \div 4 = 0$. So the slope of lines g and h are 0.

Figure 10



Vertical Lines Now that you know horizontal lines, you could surmise that vertical lines will be $X =$ some number and you are correct. Figure 11 has 2 lines, k and w. Notice the three points on each line have the same X coordinate. And every point along these lines also has the same X coordinates. The equation of line k is $X = 1$ and line w is $X = -4$.

The slope of line k is $\frac{\text{rise}}{\text{run}}$ which is $\frac{3}{0}$ between points the $(1, 0)$ and $(1, 3)$ and $3 \div 0 =$ undefined. The slope of lines k and w are undefined, because you cannot divide a number by 0. There is no answer which when multiplied by 0 will yield 3.

Figure 11

